Optimal Phosphorus Management in a Transboundary Setting: A Dynamic Game Approach

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Abstract

Phosphorus (P) runoff from agriculture is a major driver of eutrophication in transboundary water systems like Lake Erie. This paper develops a dynamic game model to examine how strategic interactions between the U.S. and Canada shape long-term crop production and environmental outcomes under stochastic soil P dynamics. The results show that while unilateral decisions often lead to higher crop production, they also result in greater environmental damage due to excessive P runoff. In contrast, incorporating transboundary nutrient spillovers naturally reduces P application and mitigates environmental harm, though at the cost of lower production. These findings suggest the importance of integrating biophysical feedback and economic incentives in nutrient management, emphasizing that long-term sustainability requires balancing productivity with environmental constraints.

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1 Introduction

Lake Erie has long been at the center of discussions on agricultural nutrient management, particularly due to its persistent phosphorus (P) pollution and the resulting harmful algal blooms (HABs) (Smith and Wilen 2003). Excessive P runoff from croplands in the U.S. and Canada has been identified as a primary driver of eutrophication, leading to deteriorating water quality, economic losses in fisheries and tourism, and increased treatment costs for drinking water (Lake Erie LaMP 2011; Environment and Climate Change Canada 2023). Despite decades of policy efforts—including voluntary conservation programs, best management practices (BMPs), and regulatory nutrient reduction targets—P runoff remains a significant challenge, exacerbated by the accumulation of P in soils (Carpenter 2008). The complexity of the Lake Erie case explains the need for dynamic and strategic approaches to P management that account for both long-term nutrient accumulation and transboundary externalities (Brock and Xepapadeas 2010).

While previous studies have examined the economic and environmental trade-offs of P reduction strategies, most rely on static models or single-agent decision frameworks, which fail to capture the strategic interactions between agricultural producers in different jurisdictions (Karp and Zhang 2006). Because P pollution is a transboundary issue, optimal management requires coordinated decision-making between the U.S. and Canada to internalize the spillover effects of nutrient runoff. In the absence of such coordination, unilateral policies may lead to inefficient outcomes, where one country's efforts are offset by the continued externalities imposed by the other (Hoel 1991).

This study develops a dynamic game-theoretic model to analyze optimal P fertilizer application strategies in a transboundary agricultural system, with Lake Erie serving as a motivating case (Xabadia et al. 2008). The model considers the strategic behavior of farmers in the U.S. and Canada, incorporating stochastic soil P dynamics, economic trade-offs between crop production and environmental damage, and cross-border nutrient spillovers (Horan et al. 2019). By solving a Markov decision process (MDP) in a dynamic game setting, we examine how different policy instruments—including fertilizer taxes, subsidies, and application caps—influence long-term environmental and economic outcomes.

A key question addressed in this study is whether unilateral policies—where each country regulates P application independently—can approximate the outcomes of a binationally optimized approach, in which both countries internalize transboundary externalities (Folmer and v Mouche

1994). The findings reveal that while aggressive unilateral interventions (e.g., high fertilizer taxes) can reduce environmental damage, they often come at the cost of reduced crop yields. Conversely, binational coordination accounts for transboundary nutrient spillovers, leading to a more efficient allocation of P fertilizer that reduces environmental damage. However, this also results in lower P application levels compared to unilateral decisions, which may come at the cost of reduced crop production.

To better understand the long-term and cross-border impacts of fertilizer use, we incorporate the concept of marginal user cost (MUC) into our analysis. Unlike typical resource models, fertilizer use increases the soil stock, making the MUC a shadow cost of future environmental degradation. As U.S. legacy P rises, Canada's MUC increases disproportionately, prompting greater self-restraint despite ongoing cross-border impacts. This asymmetry highlights the need for coordinated policies to internalize transboundary nutrient externalities.

By integrating economic incentives, strategic interactions, and transboundary externalities, this study provides a theoretical and quantitative foundation for designing more effective P management policies in shared water systems such as Lake Erie (Gren 2001). The findings explain the importance of cooperative nutrient regulation, focusing on the fact that a combination of policy interventions and technological advancements may be necessary to achieve long-term environmental sustainability without compromising agricultural productivity.

The remainder of the paper is organized as follows. Section 2 provides the background on P pollution in Lake Erie, discussing the role of agricultural runoff and transboundary externalities. Section 3 introduces the dynamic game-theoretic framework, outlining the Bellman equation and the stochastic evolution of soil P levels. Section 4 presents the yield response function estimation, using empirical data from Ohio to quantify the relationship between P fertilizer application and crop yields. Section 5 presents the simulation results, analyzing the economic and environmental trade-offs of different P management policies under unilateral and binational settings. Finally, Section 6 concludes with a discussion on our findings.

2 Background

The Lake Erie basin, straddling the border between the U.S. and Canada, presents a significant environmental management challenge due to P loading, which has profound impacts on water quality, aquatic ecosystems, and economic activities (Lake Erie LaMP 2011; Downing et al. 2021).

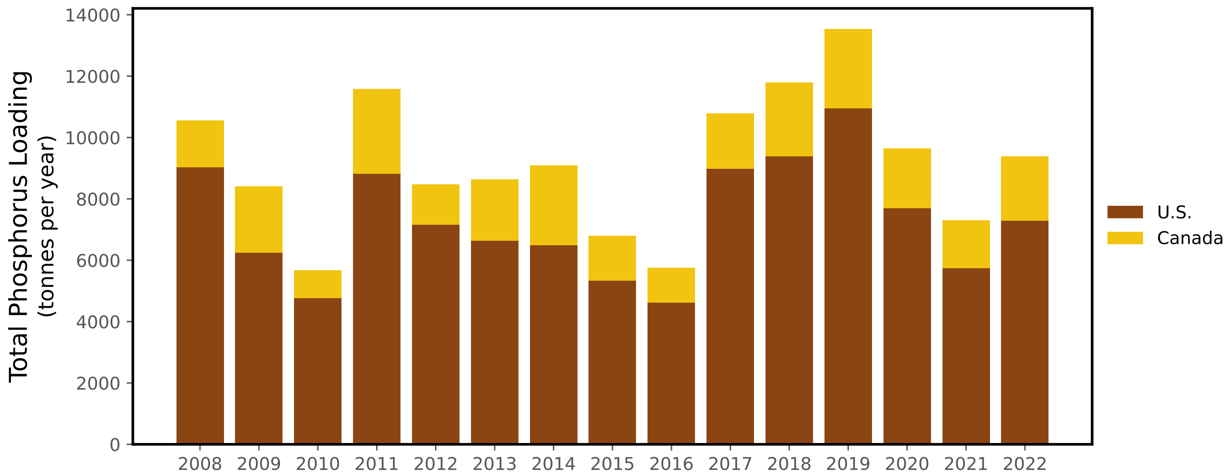

Figure 1: Total phosphorus loading to Lake Erie. The data, sourced from Environment and Climate Change Canada (2023): Canadian Environmental Sustainability Indicators, illustrates the annual P loading into Lake Erie from 2008 to 2022, distinguishing contributions from the U.S. and Canada. The United States consistently accounts for the majority of P loading, contributing over 75% of the total load annually.

Phosphorus is an essential nutrient for plant growth, but when introduced in excessive amounts into freshwater systems, it accelerates eutrophication, leading to the proliferation of HABs (Arrow et al. 2004; Conley et al. 2009; Paudel and Crago 2021; Vasseghian et al. 2024). These blooms can produce toxins harmful to aquatic life, degrade drinking water supplies, and contribute to hypoxic zones (low-oxygen areas) that threaten fish populations and biodiversity.

The sources and magnitudes of P loading to Lake Erie vary across time, space, and jurisdiction, making effective management particularly complex (Scavia et al. 2014; Maccoux et al. 2016). As shown in Figure 1, total P loading to Lake Erie exhibits substantial interannual variability. The U.S. consistently contributes a larger share of total P inputs compared to Canada, with peak loading years. This binational disparity in P contributions has important policy and economic implications. Since Canada contributes a smaller share of total P loading, unilateral mitigation efforts by Canada alone would be costly and inefficient, yielding limited environmental improvements unless matched by reductions in the U.S. watershed. The transboundary nature of P pollution means that any successful and economically efficient reduction strategy should involve coordinated efforts between the two nations to avoid cost asymmetries and ensure that the benefits of P reductions are shared equitably. The Great Lakes Water Quality Agreement (GLWQA) provides a framework for such collaboration, setting joint P reduction targets to prevent the burden from falling disproportionately on one country (Loadings and Blooms 2014).

A key challenge in reducing P loading to Lake Erie is the dominance of non-point sources,

Figure 2: Annual average (2013–2022) phosphorus loading patterns and source contributions. The data, sourced from Environment and Climate Change Canada (2023): Canadian Environmental Sustainability Indicators. Figure 2 shows the total P loading into Lake Erie (2008–2022) from multiple sources. Point sources refer to P discharges from municipal sewage treatment plants and industrial effluent, whereas non-point sources primarily stem from agricultural activities and urban stormwater runoff. Atmospheric deposition involves phosphorus settling directly into the lake from the air (Environment and Climate Change Canada 2023).

which account for 77% to 90% of total P inputs across all basins, as shown in Figure 2. While the western basin experiences the highest P loading, non-point sources—mainly from agriculture—are the largest contributors across the western, central, and eastern basins (Environment and Climate Change Canada 2023). Point and atmospheric sources play a relatively minor role, making non-point source management the primary focus for reduction efforts.

Given that non-point source (agriculture) are the primary contributor to P pollution and that P management is inherently a binational challenge, any effective reduction strategy must address both farmers' decision-making and cross-border policy coordination. Since P pollution does not adhere to political boundaries, unilateral efforts are often inefficient and costly, requiring strategic interactions between the U.S. and Canada to achieve shared reduction goals. At the same time, the effectiveness of P mitigation hinges on how farmers adjust their fertilizer use and conservation adoption over time in response to environmental conditions, economic incentives, and policy interventions. Unlike static regulatory approaches, which assume fixed behavioral responses, P management requires a dynamic framework that captures the interactions between policymakers, farmers, and environmental processes across both temporal and spatial scales.

This complexity makes a dynamic game model particularly relevant, as it allows us to analyze how strategic behavior among stakeholders evolves over time. By incorporating economic incentives, uncertainty, and transboundary externalities, the model provides insights into optimal policy coordination between the U.S. and Canada while considering the adaptive nature of agricultural decision-making. In the next section, we develop a dynamic game-theoretic framework to assess how fertilizer application choices, conservation adoption, and regulatory interventions interact, ultimately shaping long-term P loading in Lake Erie.

3 Model

This section introduces the dynamic game model governing soil P accumulation on agricultural land and the resulting economic damages due to soil P runoff. The model captures key processes of the dynamic game modeling approach for the optimal management of soil P, including the carry-over dynamics of soil P, the economic implications of P runoff on farm-level profits, and the stochastic nature of P accumulation and depletion. We first present the transition function of soil P and then extend it to address the resulting runoff and associated economic damages.

3.1 Stochastic soil phosphorus dynamics

We consider a set of farmers denoted by Ψ, each managing their P fertilization strategies. Specifically, for any farmer $i \in \Psi$, the model follows their decisions on the application of P fertilizer over time. The evolution of the stock of soil P, l_{it}, on farmer i's land per hectare at time t, is governed by a dynamic equation that captures both deterministic and stochastic elements. The formulation in this paper builds on the model of Cho et al. 2025.

The evolution of soil P for farmer i is described as:

$$l_{it+1} = \eta_t \left(1 - r_i\right) l_{it} + \left(\delta_1 + \delta_2 l_{it}\right) \underbrace{\left[f_{it} - \overbrace{\left(\delta_3 \log(l_{it}) + \delta_4\right)}^{\text{Concentration on Yield}} y\left(l_{it}, f_{it}\right) \right]}_{\text{Soil P Surplus}} \tag{1}$$

where l_{it} is the stock of soil P at time t, r_i is the P runoff rate to surface water from farm i, and η_t is the stochastic carry-over rate, governing the proportion of soil P that persists from period t to $t+1$. f_{it} represents the amount of P fertilizer applied by farmer i at time t. $y(L_{it}, F_{it})$ is the crop yield function, which depends on both soil P l_{it} and applied P fertilizer f_{it}. $(\delta_1 + \delta_2 l_{it})$ captures the response of soil P surplus to the initial stock level and the rate of P application (Ekholm et al. 2005).

The expression inside the brackets represents the soil P surplus, which is the difference between the P applied through fertilizer f_{it} and the P uptake by crops. Crop uptake is modeled by a yield response function $y(l_{i,t}, f_{i,t})$ scaled by a diminishing return term $(\delta_3 \log(l_{i,t}) + \delta_4)$. This diminishing return effect reflects well-documented agronomic principles, whereby the marginal productivity of P in crop yield decreases as Soil P accumulates (Myyrä et al. 2007; Fulford and Culman 2018; Culman et al. 2023).

The soil P carry-over rate η_t contributes to stochastic motions in this dynamic model. It is formulated to capture the uncertainty in P retention or depletion between periods, and it incorporates both deterministic and stochastic components. Specifically, we model η_t as:

$$\eta_t = \exp\left[\mu_\eta - \frac{s_\eta^2\left(l_{it}\right)}{2} + s_\eta\left(l_{it}\right)\omega_t\right] \tag{2}$$

where μ_η represents the log mean rate of change in soil P, which reflects the natural rate of P retention or decay in the soil. $s_\eta(l_{it})$ is the standard deviation of the log percentage growth rate of soil P, which is modeled as a function of the current stock l_{it}. ω_t is a normally distributed shock term $(\omega_t \sim \mathcal{N}(0,1))$, which introduces randomness into the carry-over rate, representing external factors such as weather conditions, soil characteristics, or management practices that affect P retention.

The parameter μ_η is assumed to be negative, reflecting the fact that, in the absence of further P application or crop uptake, soil P is expected to decay over time (Ekholm et al. 2005; Iho and Laukkanen 2012). However, this decay process is stochastic, as represented by the inclusion of the standard deviation term $s_\eta(l_{it})\omega_t$. This stochastic component acknowledges that soil P dynamics are influenced by factors beyond the farmer's control, such as soil type, precipitation patterns, and other environmental variables.

The variance of the carry-over rate is specified as:

$$s_\eta^2\left(l_{it}\right) = \ln\left(1 + \frac{\sigma^2}{l_{it}^2 \cdot \mathbb{E}\left[\eta_t \mid l_{it}\right]^2}\right) \tag{3}$$

where σ^2 is an uncertainty variance, and $\mathbb{E}[\eta_t \mid l_{it}]$ represents the expected carry-over rate conditional on the current stock of soil P. This formulation ensures that the variance of the next-period soil P stock remains bounded as l_{it} accumulates, [1] preventing unrealistic behavior where the uncertainty

[1] The expression $\mathbb{E}[\eta_t \mid l_{it}] = \exp(\mu_\eta)$ represents the expected carry-over rate of soil P conditional on the current stock l_{it}. Given the log-normal specification of η_t and assuming the shock term $\omega_t \sim \mathcal{N}(0,1)$, we can derive the expected value of η_t by taking the conditional expectation with respect to ω_t. Since the expectation of the exponential of a normal random variable ω_t is given by $\exp(s_\eta^2(l_{it})/2)$, the stochastic term cancels out with the variance adjustment term $s_\eta^2(l_{it})/2$, leaving $\mathbb{E}[\eta_t \mid l_{it}] = \exp(\mu_\eta)$. This result implies that the expected value of the carry-over rate is determined solely by the log mean growth rate μ_η, while the variance $s_\eta^2(l_{it})$ introduces uncertainty around this mean, capturing the effects of stochastic shocks.

would grow without bound for large l_{it}. Such a specification follows well-established approaches in modeling environmental stocks under uncertainty (Loury 1978, Gilbert 1979; Melbourne and Hastings 2008, Sims et al. 2017; Sloggy et al. 2020).

The introduction of stochasticity in the carry-over process captures real-world complexities where P retention and depletion are not deterministic processes. Factors such as variations in soil composition, temperature, moisture, and microbial activity contribute to the stochastic nature of P dynamics, which this model seeks to represent. By introducing a stochastic component into the P carry-over, the model can better account for observed variabilities in soil P stocks across farms and over time.

3.2 Dynamic game formulation and equilibrium

The strategic interactions between countries Ψ are captured through a Markov perfect equilibrium, which accounts for the fact that each farmer's decision impacts not only their own payoff but also the runoff and associated damages that affect both players. This approach allows us to study the externalities arising from P runoff and how these externalities influence the optimal management of P fertilization.

The annual payoff of country $i \in \Psi$ is evaluated as the profit generated by crop yields minus the cost of P fertilizer and the damages incurred due to soil P runoff. Formally, the expected per-hectare profit for country i is expressed as:

$$\pi_i\left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}, f_{it}\right) = p^y_{it} y_{it}\left(l_{it}, f_{it}\right) - p^f_{it} f_{it} - d_i\left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}\right) \tag{4}$$

where $\Psi_i = \Psi \setminus \{i\}$ indicates the population Ψ excluding country i. p^y_{it+1} is the price of the crop, p^f_{it} is the price of the P fertilizer, and $y_{it}(l_{it}, f_{it})$ represents the crop yield as a function of the soil P stock l_{it} and the current P fertilizer application f_{it}.

The last term, $d_i\left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}\right)$, represents the damage function, which models the economic damages due to P runoff from both country i's farm and other neighboring country's farms j. Many studies in the literature define the damage function as a linear relationship where environmental (eutrophication) damage is proportional to P runoff, typically expressed as a constant marginal damage parameter multiplied by the amount of soil P runoff (Smith et al. 1995; Sharpley et al. 1996; Iho and Laukkanen 2012, Tang 2018). This approach assumes that each additional unit of P runoff causes the same incremental increase in damage, without accounting for potential threshold effects. However, empirical evidence suggests that eutrophication damage often exhibits nonlinear patterns, where small increases in P runoff may have minimal effects at low concentrations but lead to severe ecological damage once critical thresholds are exceeded (Carpenter et al. 1999; Jarvie et al. 2013b, and Schindler et al. 2016). To better capture these dynamics, we define the damage function to a power function with elasticity, given by:

$$d_i(l_{it}, \{l_{jt}\}_{j\in\Psi_i}) = c \cdot \left(r_i l_{it} + \sum_{j\neq i} \tau_j r_j l_{jt} \right)^{\rho} \quad (5)$$

where r_i and r_j represent the runoff rates of soil P from countries i and j, respectively, and c is the constant marginal damage of P loading. τ is the weight reflecting differences in P transport efficiency between regions. To capture the uncertainty in P transport efficiency between regions, we model $\tau_j \sim Beta(\alpha_j, \beta_j)$, as the Beta distribution is well-suited for variables bounded between 0 and 1 and allows for flexible shapes depending on the parameterization. This damage function allows for flexible responses through ρ, where $\rho > 1$ captures threshold effects and $0 < \rho < 1$ reflects diminishing marginal damage.

In the dynamic game setting with multiple countries, the strategic decisions are made over time, considering not only the current payoff but also future consequences of P runoff and its cumulative effects on the environment. Each country optimizes their fertilizer application strategy by weighing the immediate benefits of increased crop yields against the future costs of environmental degradation caused by P runoff. These intertemporal trade-offs are captured by a Markov perfect equilibrium (MPE), where each country's strategy depends only on the current state of the system—specifically, the soil P stocks on both countries, l_{it} and l_{jt}.

The equilibrium concept of the MPE assumes that both countries are noncooperative and forward-looking and that their actions take into account not only the current conditions but also the expected future actions of the other country (Gollier and Treich 2003; Miranda and Fackler 2004; Hovi et al. 2015). The problem is inherently dynamic because the decisions made by each country at time t affect the soil P stock in future periods, which in turn impacts future crop yields and environmental damages. This leads to a situation where both countries must strategically anticipate the other's actions, given the shared nature of the runoff-induced damage.

The optimization problem for each country is framed through the Bellman equation, which represents the recursive nature of the decision-making process. For country i, the value function $V_i\left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}\right)$ reflects the maximum expected net present value (ENPV) of their profits over time, given the current state of soil P stocks on both countries. The Bellman equation for country i is formulated as follows:

$$V_i\left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}\right) = \max_{f_{it}\in\left[0,\bar{f}\right]} \left\{ \begin{array}{l} \mathbb{E}_{\tau_j}\left[\pi_i\left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}, f_{it}\right)\right] \\ +\beta\mathbb{E}\left[V_i\left(l_{it+1}, \{l_{jt+1}\}_{j\in\Psi_i}\right) \mid l_{it}, \{l_{jt}\}_{j\in\Psi_i}, f_{it}, \{f_{jt}\}^*_{j\in\Psi_i}\right] \end{array} \right\} \tag{6}$$

where the β is the discount factor and the expectation $\mathbb{E}[\cdot]$ represents the uncertainty in future outcomes, conditional on both countries' current decisions.

The dynamic nature of the problem stems from the fact that each country's decision at time t affects the future states of soil P stocks on both countries, l_{it+1} and l_{jt+1}. Moreover, since P runoff creates externalities that affect both farmers, the value function depends on the current and future decisions of the other country. The term $\{f_{jt}\}^*_{j\in\Psi_i}$ represents the optimal fertilizer application policy of country j, assuming they are also acting optimally given the current state. This interdependence between each country's decisions is central to the MPE, where each country's strategy is the best response to the other's actions in each period.

To solve for the equilibrium strategies, the system of Bellman equations for the countries must be solved simultaneously. The solution yields the optimal P application policies for both countries, f^*_{it} and f^*_{jt}, which specify the optimal amount of fertilizer to apply in each period, given the current soil P stocks on both countries. These strategies balance the trade-offs between the short-term benefits of increased crop yields and the long-term costs of P runoff.

The MPE ensures that the strategies of both countries are mutually consistent, meaning that neither country has an incentive to deviate from their equilibrium strategy, given the strategy of the other. This equilibrium captures the strategic interdependence between the countries, as each country internalizes the externality caused by P runoff. By following their equilibrium strategies, the countries contribute to managing P runoff in a way that considers not only their own profits but also the broader environmental impacts on the shared ecosystem.

3.3 Marginal User Cost

To examine the intertemporal tradeoffs embedded in fertilizer use decisions, we derive the marginal user cost (MUC) from the Bellman equation (6) (Cho et al. 2024). This term captures the shadow value associated with increasing soil P today—namely, the effect of a marginal rise in current P application on the present value of future payoffs, via its influence on soil P accumulation.

Differentiating the equation (6) with respect to the fertilizer decision f_{it} yields:

$$\frac{dV_i(l_{it}, \{l_{jt}\}_{j\in\Psi_i})}{df_{it}} = 0 = \left[\underbrace{\frac{\partial \mathbb{E}_{\tau_j}\left[\pi_i(l_{it}, \{l_{jt}\}_{j\in\Psi_i}, f_{it})\right]}{\partial f_{it}}}_{\text{current-period marginal profit}} + \underbrace{\beta \mathbb{E}\left[\frac{\partial V_i(l_{it+1}, \{l_{jt+1}\}_{j\in\Psi_i})}{\partial l_{i,t+1}} \cdot \frac{\partial l_{it+1}}{\partial f_{it}}\right]}_{\text{marginal user cost (MUC}_{it})} \right]_{f_{it}=f^*_{it}} \tag{7}$$

The first term captures the current-period marginal profit of fertilizer-—namely, the increase in revenue from higher yields net of the fertilizer cost. The second term, the MUC, represents the discounted shadow value of how today's fertilizer decision affects future soil P accumulation and, in turn, future payoffs. The inner components of the MUC highlight its economic interpretation. $\partial V_i(l_{i,t+1}, \{l_{j,t+1}\}_{j\in\Psi_i})/\partial l_{i,t+1}$ measures the shadow value of soil P in the next period. This reflects both productive benefits—higher soil fertility improves yields—and environmental costs—higher stocks increase runoff and eutrophication damages. Next, $\partial l_{i,t+1} \partial f_{it}$ captures the extent to which current fertilizer use contributes to future soil P accumulation. Unlike classical resource extraction models, where use reduces the stock, here the control variable adds to the state: fertilizer application raises soil P, with persistent legacy effects (Sharpley et al., 2013; Jarvie et al., 2013a). This inversion implies that the MUC reflects the opportunity cost of building up an excessive stock rather than depleting a scarce one.

Taken together, the MUC thus reflects the net shadow value of an additional unit of soil P created today: positive if the accumulation improves future productivity more than it increases damage, and negative if further accumulation mainly exacerbates runoff externalities.

Rearranging this condition provides an intuitive equilibrium condition:

$$p_y \cdot \frac{\partial y(l_{it}, f_{it})}{\partial f_{it}} = p_f - \text{MUC}_{it} \tag{8}$$

where the left-hand side is the marginal revenue product of fertilizer and the right-hand side is the fertilizer price net of the shadow cost. The presence of the MUC implies that the effective cost of fertilizer faced by the farmer is not merely the market price p_f, but rather $p_f - \text{MUC}_{it}$. In other words, the MUC operates like an implicit shadow tax that adjusts private marginal incentives to reflect intertemporal consequences.

The inclusion of the MUC also highlights an important distinction between unilateral and binational decision-making. A unilateral farmer, when solving the Bellman equation, internalizes only the shadow value of changes to their own soil P stock, while ignoring the transboundary spillovers of P accumulation and the associated environmental damages that fall on the neighboring country. Formally, this implies that the unilateral MUC is computed solely from $\partial V_i(l_{i,t+1})/\partial l_{i,t+1}$, without accounting for $\{l_{j,t+1}\}_{j\in\Psi_i}$. By contrast, under binational coordination, the MUC incorporates both the domestic and transboundary components of soil P, so that the shadow tax term reflects the full marginal opportunity cost of P accumulation.

This asymmetry has a clear implication: the MUC faced by a unilateral farmer will generally be larger in magnitude than the socially efficient MUC, because the unilateral decision-maker disregards external damages imposed on the other jurisdiction. As a result, the fertilizer rate chosen unilaterally exceeds the binationally optimal rate, leading to systematic over-application. In equilibrium, the gap between unilateral and binational fertilizer use is precisely explained by the omission of transboundary damages in the unilateral farmer's MUC calculation.

4 Empirical model specification for yield response

4.1 Data description

In this section, we outline the empirical model used to estimate the yield response function y_{it} that analyzes the impact of P fertilizer application and soil P on corn yield. The data used in this estimation originate from long-term field trials in Ohio (Clark, Wayne, and Wood counties) assessing P fertilizer application and their effect on crop yield given soil P levels. Specifically, we focus on the field trials reported in Culman et al. (2023) Dataset 2, covering 16 years of experiments (2006-2021) at three research farms in Ohio. The trials employed a randomized complete block design with three P application rates: an unfertilized control ($0\times$), an estimated crop removal rate ($1\times$), and an excessive application rate ($2-3\times$ the removal rate) [2] Soil samples were analyzed

[2] During the initial phase of the experiment (2006-2014), the estimated crop removal rate for P fertilizer was P_2O_5 60.1kg/ha, based on the estimated removal rate of 2005 Ohio (Vitosh et al. 1995, Fulford and Culman 2018). The field trials consider $2\times$ the removal rate for excessive application cases during this period (Fulford and Culman 2018). However, Fulford and Culman (2018) found that actual removal rates exceeded these estimates. Consequently, from

Experimental location	Variable	Obs	mean	Std dev	min/max
Clark	Corn yield (Mg/ha)	30	10.21	2.14	6.2/12.9
	P application (kg/ha)	30	122.66	128.99	0/336.3
	Soil P (mg/kg)	30	20.25	7.88	9.8/40
Wayne	Corn yield (Mg/ha)	36	11.17	2.71	6/15
	P application (kg/ha)	36	127.13	131.29	0/336.3
	Soil P (mg/kg)	36	16.37	9.09	4.3/37.3
Wood	Corn yield (Mg/ha)	48	9.54	2.22	6/14.3
	P application (kg/ha)	48	115.95	123.74	0/336.3
	Soil P (mg/kg)	48	21.28	7.67	11.9/39
Total	Corn yield (Mg/ha)	114	10.23	2.45	6/15
	P application (kg/ha)	114	121.25	126.49	0/336.3
	Soil P (mg/kg)	114	19.46	8.40	4.3/40

Table 1: Summary statistics for Ohio field trials data. The experimental location level is county in Ohio.

before planting to determine baseline Mehlich-3 extractable P levels, and crop yield data were recorded after harvest.

For our estimation, we focus exclusively on trials where P fertilizer application resulted in a statistically significant yield increase, excluding non-responsive cases. The dataset thus reflects only instances where P fertilizer had a positive impact on crop yield, ensuring that our estimates capture the actual effect of P application rather than noise from non-significant responses.

Table 1 presents the summary statistics for the Ohio field trial dataset, which includes observations from three experimental locations. The variables reported are corn yield (Mg/ha), P fertilizer application (kg/ha), and soil P concentration (mg/kg). The average corn yield across all sites is 10.23 Mg/ha, with values ranging from 6 to 15 Mg/ha. The mean P application rate is 121.25 kg/ha, with a standard deviation of 126.49 kg/ha, which appears relatively large due to the experimental design. Since this dataset originates from a controlled field experiment with discrete P application treatments (0×, 1×, and 2–3× the estimated crop removal rate) rather than a continuous distribution of fertilizer use, most observations cluster around these predetermined levels rather than being evenly spread across the range. This results in a high standard deviation, as the experimental setup

2015-2021, the fertilizer application rates were adjusted to 112.1kg/ha (1×), and the experiment considered 336.3kg/ha for excessive application cases (3×).

	Log Corn Yield (Mg/ha)
f_{it}	0.0025^{***}
	(0.0001)
$\ln(l_{it})$	0.1492^{*}
	(0.0391)
f_{it}^2	-0.0000002^{***}
	(0.000001)
$f_{it} \times \ln(l_{it})$	-0.0003^{**}
	(0.00005)
Const.	1.1959^{**}
	(0.1612)
Location Fixed Effects	Yes
Time Fixed Effects	Yes
Observations	114
Adjusted R^2	0.7810

Table 2: Corn yield response estimation. Standard errors are clustered at the plot level c, and clustered standard errors are reported in parentheses. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

includes both unfertilized plots and excessively fertilized treatments to capture the full range of P fertilizer effects on yield.

4.2 Estimation framework

To quantify the yield response, we specify the following log-linear model:

$$\ln(y_{ict}) = \beta_0 + \beta_1 f_{ict} + \beta_2 \ln(l_{ict}) + \beta_3 f_{ict}^2 + \beta_4 f_{ict} \ln(l_{ict}) + u_i + \nu_t + \epsilon_{ict}^y \tag{9}$$

where $\ln(y_{ict})$ denotes the natural logarithm of yield for plot c in experimental location i and year t. u_i is the experimental location fixed effect and ν_t is the time fixed effet. The estimation results are in Table 2. The regression results in Table 2 indicate that fertilizer application has a positive and diminishing marginal effect on yield. The coefficient on fertilizer, f_{it}, is positive and significant, while the negative and highly significant coefficient on its squared term, f_{it}^2, implies concavity of the yield response function. This pattern suggests that yield increases with fertilizer at a decreasing rate, which is consistent with standard agronomic production theory (Berck et al., 2000; Myyrä et al., 2007; Tembo et al., 2008; Fulford and Culman, 2018).

Soil P, measured by $\ln(l_{it})$, also has a significant and positive effect, indicating that higher soil P stocks are associated with higher yield levels. The interaction term, $f_{it} \times \ln(l_{it})$, is negative and statistically significant, suggesting that the marginal productivity of applied fertilizer declines when soil P is already high.

In our estimation, standard errors are clustered at the plot level c, corresponding to the three fertilizer treatments. Clustering at this level is motivated by the experimental design: observations within the same treatment category may share unobserved agronomic or management similarities that induce correlated residuals. For example, plots receiving the same relative rate of fertilizer are likely to exhibit similar yield responses conditional on soil P. Ignoring such intra-group correlation would lead to underestimated standard errors and inflated significance levels. By clustering at the treatment level, we obtain conservative inference that is robust to arbitrary correlation of errors within treatment groups across counties and years.

Taken together, these estimates highlight the complementarity and substitutability between fertilizer application and soil P: while both inputs contribute positively to yield, the return to one input diminishes as the other increases. The inclusion of location fixed effects (u_i) and time fixed effects (ν_t) ensures that unobserved heterogeneity across experimental sites and common temporal shocks are properly accounted for.

We employ this yield response function uniformly for both the U.S. and Canada in the dynamic game framework. This modeling choice ensures that differences in fertilizer application across the two countries are not attributed to underlying disparities in production technology but arise solely from their strategic interactions and the transboundary effects of P runoff. By holding the yield response function constant, we isolate the role of soil P dynamics and cross-border externalities in shaping optimal fertilizer decisions.

5 Results

5.1 Optimal phosphorus application

In our analysis of the Lake Erie case, we simplify the farmer population Ψ to two groups: $\Psi = \{\text{U.S., Canada}\}$. The other parameter values are summarized in Table A1. Given the interconnected nature of agricultural markets and trade between the U.S. and Canada, we assume that both countries share the same prices for P fertilizer and corn. This assumption helps isolate the effects of P runoff dynamics rather than confounding them with price differences, allowing the model to focus on the strategic interactions between farmers in managing P application and the resulting transboundary environmental impacts.

Figure 3 presents the optimal P application policies for the U.S. and Canada under different soil P conditions and transboundary interactions. The results compare unilateral policies, derived using stochastic dynamic programming (SDP), where each country assumes no transboundary P effect, with binational policies, which account for P spillover across borders. The analysis highlights the inefficiencies in unilateral decision-making and the extent of P misallocation, which exacerbates environmental externalities and results in significant welfare losses for both agricultural producers and environmental stakeholders.

The unilateral P application policy represents the optimal strategy when each country assumes that the other does not contribute to transboundary P levels. In other words, the model optimize P application under the assumption that transboundary P contribution remains constant at zero. This assumption leads to policies that focus solely on domestic soil P levels, disregarding the impact of cross border nutrient flow. However, unilateral policies in shared environmental systems often lead to policy myopia, where short-term gains in productivity come at the cost of long-term environmental degradation.

Conversely, the binational P application policy accounts for the interaction between U.S. and Canada agricultural runoff. When both countries recognize the contribution of transboundary P, the optimal P application rates adjust accordingly, leading to lower application levels as transboundary P increases. The consideration of shared P loads ensures that each country internalizes the externalities of its P use, leading to more environmentally sustainable outcomes. Additionally, this lower application pattern increases when the country has higher domestic P levels.

An important finding of our analysis is the presence of P misallocation (shaded region in Figure 3), where P application under unilateral policies deviate from the binationally optimal levels. This misallocation arises because unilateral policies ignore transboundary P contributions, leading to over application of P fertilizer relative to the socially optimal level. Over-application not only reduces the economic efficiency of fertilizer use but also increases the likelihood of policy intervention in the form of stricter environmental regulations.

For instance, as transboundary P contribution increases, a country adhering to a unilateral policy continues to apply P at the same rate, whereas the binational approach would dictate a reduction in application. This failure to adjust results in excessive P inputs, further contributing to P loading in shared water bodies (e.g., Lake Erie), increasing the risk of eutrophication and HABs. The literature on environmental spillovers suggests that misallocated resources in transboundary

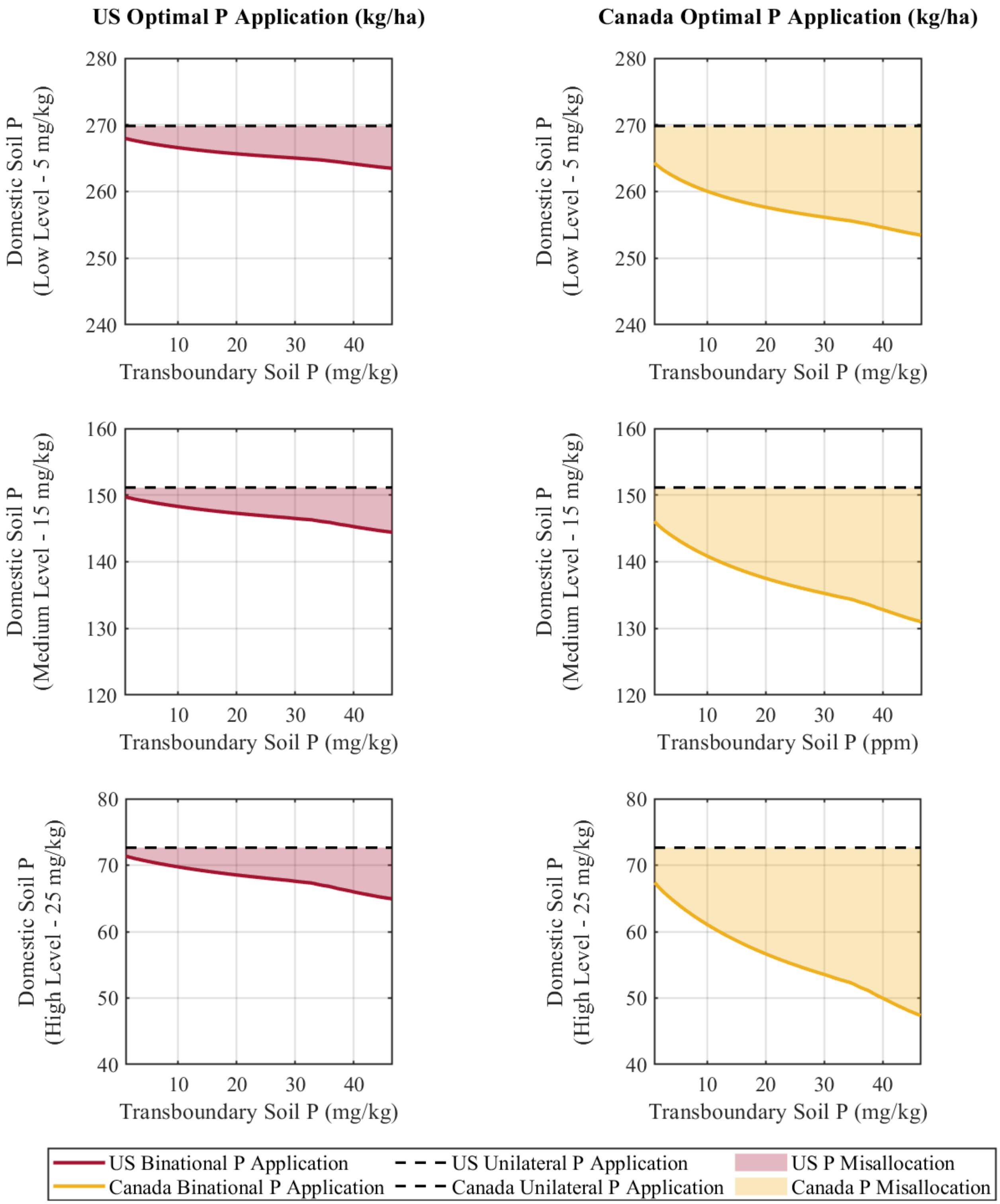

Figure 3: Optimal phosphorus application under domestic and transboundary soil P levels. Domestic soil P level refers to the P concentration within a country's own farmland, affecting its fertilizer needs. Transboundary soil P level represents P levels in a neighboring country, which can influence optimal fertilizer application. For the US, the domestic soil P level (rows) refers to P within the US, while the transboundary soil P level (x-axis) represents P levels in Canada.

pollution settings often generate deadweight losses, where both nations suffer greater costs than necessary due to inefficient policy design (Phaneuf and Requate 2016). Our findings shows the need for cooperative P management policies between the US and Canada to mitigate the environmental consequences of misaligned agricultural practices.

5.2 Crop production and environmental damage dynamics

Figure 4 presents the long-term evolution of corn production and environmental damage under different P management for the U.S. and Canada. The figure shows differences between U.S. and Canada, as well as the implications of unilateral versus binational P management approaches.

The first row of Figure 4 compares corn production and environmental damage in the U.S. and Canada under binational optimal P application and soil P dynamics. One distinction is that the U.S. receives relatively less external P inflow from Canada than vice versa, due to the directional nature of P runoff as depicted in Figure 1. Because of this asymmetric flow, the U.S. experiences lower external environmental damage from cross-border P spillovers, making the marginal cost of additional P application appear lower. This incentivizes U.S. farmers to apply more P fertilizer, leading to higher soil P levels and greater crop yields compared to Canada. However, due to the directional flow of P runoff, Canada experiences higher long-term environmental damage, as much of the excess P applied in the U.S. This results in greater eutrophication risks and water quality degradation in Canada.

An important implication of this difference is that unilateral strategies, where each country ignores transboundary effects, disproportionately increase the environmental costs borne by Canada. Since U.S. runoff significantly affects Canada but not vice versa, unilateral U.S. policies that fail to account for transboundary nutrient spillovers result in excessive environmental degradation in Canada. This reinforces the need for coordinated binational P management to ensure more sustainable agricultural production in both countries.

The second row of Figure 4 compares total corn production and total environmental damage across both countries under unilateral and binational P application policies. In a unilateral scenario, each country maximizes its own short-term agricultural output without considering cross-border externalities. This results in higher P application levels, leading to greater crop production, as seen in Figure 4 for the unilateral scenario. However, this strategy also leads to substantial long-term environmental damage. Importantly, total environmental damage under the binational scenario is lower than under the unilateral approach, demonstrating that the binational policy suggests the

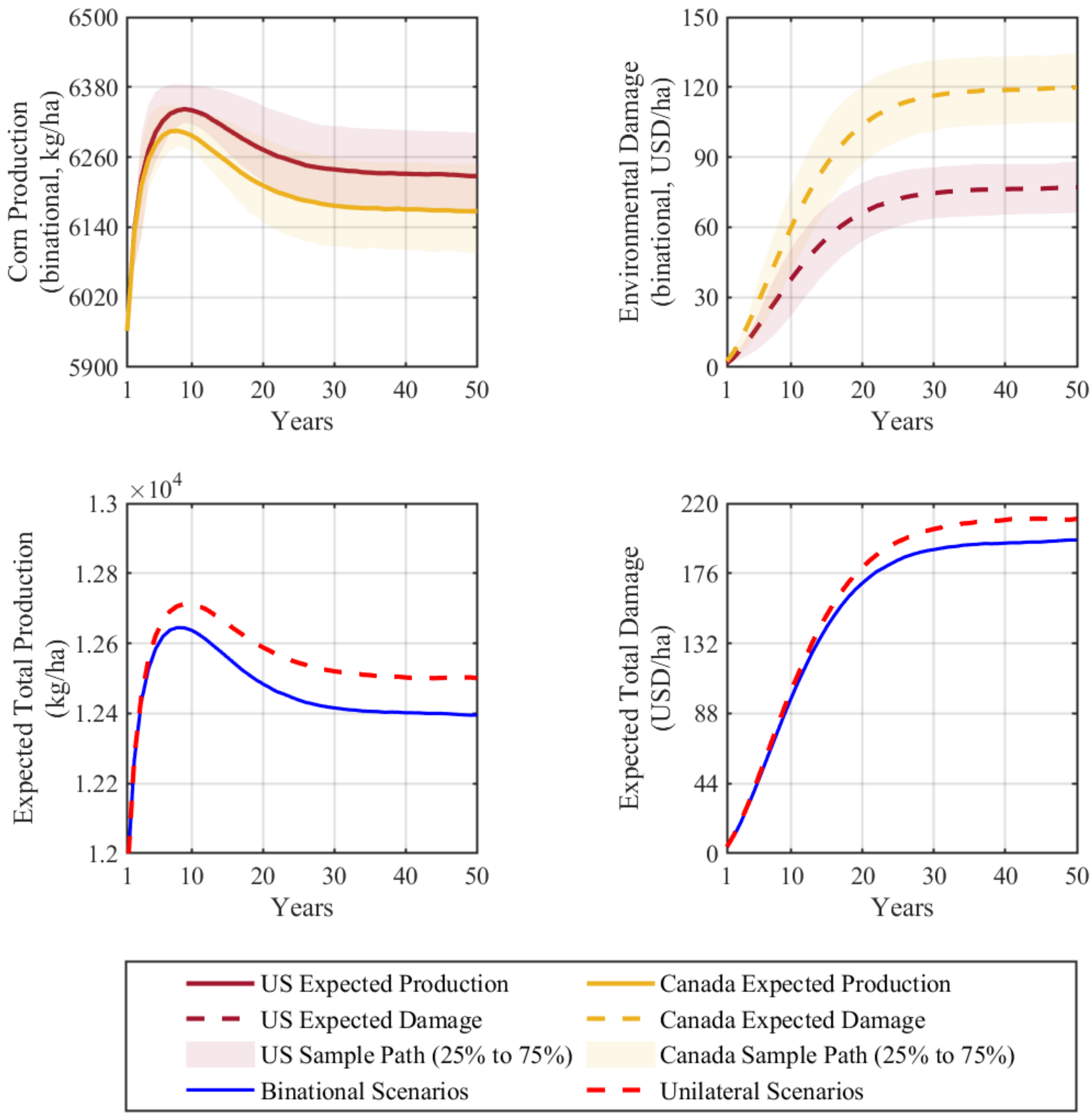

Figure 4: Example of corn production and environmental damage dynamics. Figure 4 presents the simulated trajectories of corn production and environmental damage. Total values represent the sum of the U.S. and Canada cases. The results are based on 10,000 Monte Carlo simulations, with shaded regions indicating the 25% to 75% percentile range of stochastic outcomes. The initial level of soil P for the U.S. and Canada is the minimum level (i.e., 1 mg/kg). Other initial conditions are in the Appendix.

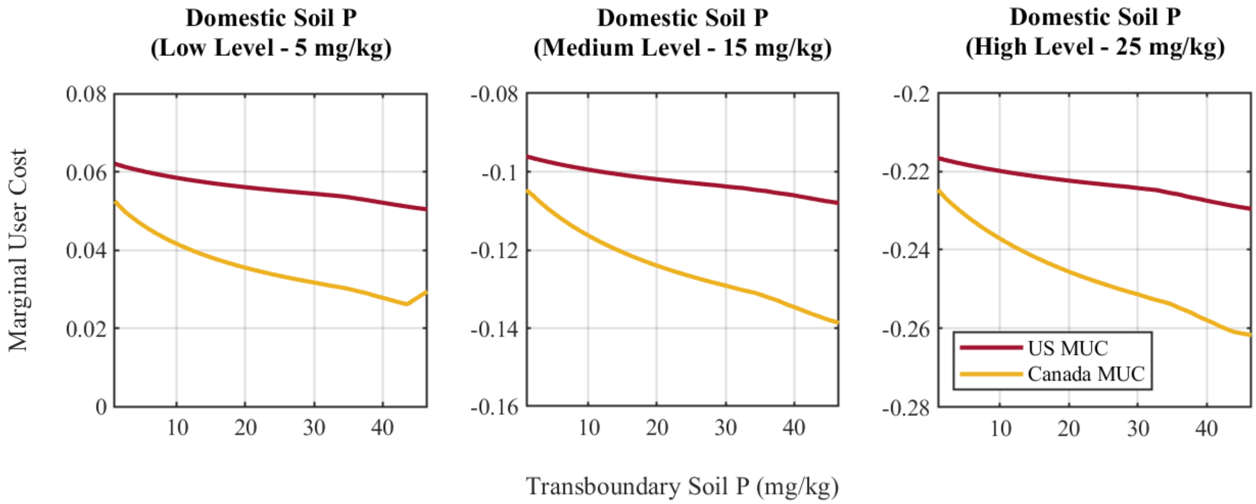

Figure 5: Marginal user cost by transboundary and domestic soil phosphorus levels. Marginal user cost is computed for fixed domestic soil P levels (5, 15, 25 mg/kg).

need to consider cross-border P spillovers in optimizing P application and mitigating environmental damage.

These results emphasize the trade-offs between productivity gains and environmental damage in transboundary agricultural systems. Unilateral P application generates more crop yields but leads to excessive environmental damage, necessitating costly regulatory interventions. These results motivate further analysis of potential policy interventions, which are explored in the following section.

5.3 Marginal User Cost and Transboundary Sensitivity

Figure 5 depicts the relationship between the MUC of P fertilizer use and transboundary soil P inflows under different levels of domestic soil P. The figure is organized into three panels, each corresponding to a fixed level of domestic soil P: low (5 mg/kg), medium (15 mg/kg), and high (25 mg/kg). The horizontal axis measures the amount of soil P entering from the neighboring country, while the vertical axis shows the computed MUC. Two lines are plotted: the red line represents the U.S. and the yellow line represents Canada. By jointly considering these results, the figure highlights how the economic shadow value of P use changes with both domestic soil conditions and cross-border nutrient flows, and how this dynamic generates asymmetries between the two countries.

When domestic soil P is low, both countries exhibit positive MUC values. This indicates that additional fertilizer use increases net future benefits. Because soils are nutrient-deficient, applying more P increases crop yields and enhances future productivity, thereby generating long-term gains.

However, as transboundary inflows of P increase, the MUC steadily declines. Importantly, external inflows do not directly increase domestic soil P stocks. Instead, they enter the damage function and magnify environmental harm. Thus, even if domestic soil conditions remain unchanged, the total damage associated with fertilizer use increases once external inflows are taken into account, reducing the net value of additional applications. Consequently, as inflows rise, the economic net benefit of fertilizer falls, and the MUC declines. Because Canada receives more P inflows from the United States, its MUC falls more sharply.

When domestic soil P reaches a medium level, the MUC becomes negative for both countries. This means that additional fertilizer use now imposes greater future costs, while the incremental yield gains are negligible. Since soils are already sufficiently fertile, further application contributes primarily to environmental damage. With external inflows added, the situation worsens. Because inflows amplify environmental damage, the MUC falls further into negative territory. In particular, Canada faces steeper declines, as U.S. inflows are more substantial. As a result, Canadian farmers have stronger incentives to reduce fertilizer use compared to U.S. farmers. However, because a significant share of damages originates from U.S. runoff, unilateral Canadian reductions cannot fully resolve the problem.

When domestic soil P is already high, the MUC is strongly negative for both countries. In this regime, additional fertilizer use primarily generates long-term environmental costs, with virtually no gains in productivity. As shown in the figure, MUC values for both countries fall below -0.2. As external inflows rise, damages intensify further. Since the U.S. receives relatively little inflow from Canada, its MUC declines only moderately. By contrast, Canada absorbs significant U.S. inflows, leading to a steeper decline in its MUC. In effect, Canadian farmers face a larger "shadow tax" on fertilizer use, making reductions unavoidable. However, because these adjustments occur only within Canada, they do not offset the damages originating from the U.S.

The figure clearly demonstrates the economic meaning of the MUC concept. The MUC is the shadow price that reflects the combined effect of current fertilizer use on future productivity and environmental damage. Positive values indicate that fertilizer use generates net social benefits, while negative values indicate that future damages outweigh private gains. The role of external inflows is particularly important. While inflows do not change domestic soil conditions directly, they increase environmental damages and thereby reduce the net value of domestic fertilizer use.

For this reason, the MUC for the U.S. declines only modestly with inflows, while Canada's MUC falls sharply due to its greater exposure to U.S. runoff. Thus, the same unit of fertilizer use translates into a larger shadow cost in Canada.

Two important policy implications emerge from these results. First, unilateral management is inefficient. Because Canada bears greater external damages, it has stronger incentives to restrict fertilizer use. However, much of the damage originates in U.S. runoff, meaning Canada's unilateral actions cannot fully mitigate the problem. This creates an imbalance where effort is high but effectiveness is limited. Second, binational cooperation is essential. Under cooperative management, both countries would internalize MUC values that include external inflows. This effectively raises the implicit shadow tax on U.S. fertilizer use, reducing excessive application and alleviating the disproportionate burden on Canada. In this sense, instruments such as fertilizer taxes, nutrient trading systems, or application caps can be interpreted as policy tools to align MUC across borders.

Figure 5 highlights the fundamental imbalance in cross-border P management. When domestic soil P is low, fertilizer use can be justified, but at medium or high levels, further use generates net social losses. External inflows exacerbate damages and lower MUC, with effects being much stronger in Canada. This places Canada in a position where it must regulate more strictly while still suffering damages from U.S. sources. Thus, efficient and equitable P management requires cooperative policies that explicitly internalize MUC values, including those driven by external inflows.

6 Policy analysis

Policies for managing P can be broadly grouped into two categories. The first category directly regulates fertilizer use by altering farmers' incentives. These measures include taxes, subsidies, or quantitative limits on P application, which directly shift the marginal cost and thereby influence optimal fertilizer decisions (Shortle and Dunn, 1986; Xabadia et al., 2008; Iho and Laukkanen, 2012).

The second category reduces P runoff indirectly through changes in production practices or biological processes. Examples include the adoption of cover crops that absorb residual nutrients, or the use of mycorrhizal fungi that enhance P uptake efficiency (Smith et al., 2011; Kaspar and Singer, 2011; Kaur et al., 2024). Such measures lower the effective P surplus in soils and reduce downstream eutrophication risk, even without explicit restrictions on fertilizer application.

In what follows, we first analyze policies that directly control fertilizer use, and then turn to

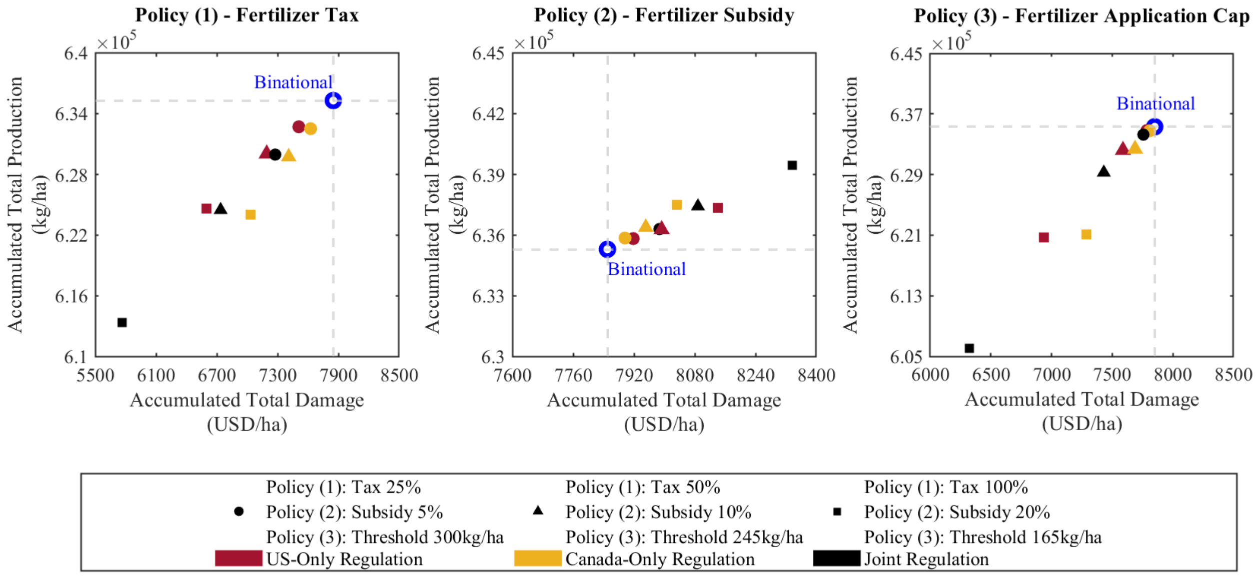

Figure 6: Accumulated production and environmental damage under P policies Figure 6 presents the accumulated total production and environmental damage over a 50-year period based on 10,000 Monte Carlo simulations(The initial level of soil P is the minimum level (i.e., 1 mg/kg)). The results are derived by computing annual averages and summing them over time. For threshold-based policies, the 300 kg/ha limit represents the 90th percentile of the maximum P application observed in our dataset (330 kg/ha), while the 245 kg/ha and 165 kg/ha thresholds correspond to the 75th and 50th percentiles, respectively.

indirect approaches aimed at mitigating P runoff.

6.1 Direct Controls on Fertilizer Application

To evaluate the economic and environmental implications of direct fertilizer regulations, we examine three commonly proposed instruments: fertilizer taxes, subsidies for reduced use, and application caps. Figures 6 and 7 present the long-run outcomes of these policies in terms of accumulated total production and accumulated environmental damage over a 50-year horizon, based on 10,000 Monte Carlo simulations.

Figure 6 reports the results when policies are implemented under binational decision-making, in which both countries jointly internalize cross-border nutrient spillovers. The panels demonstrate how each instrument—taxes, subsidies, and application thresholds—shapes the long-term trade-off between agricultural productivity and environmental damage.

In general, stricter interventions (e.g., higher tax rates or lower caps) reduce environmental damage but also lower total production. Taxes directly increase the effective fertilizer price, creating a stronger disincentive to apply P, while subsidies reduce input costs conditional on adopting lower

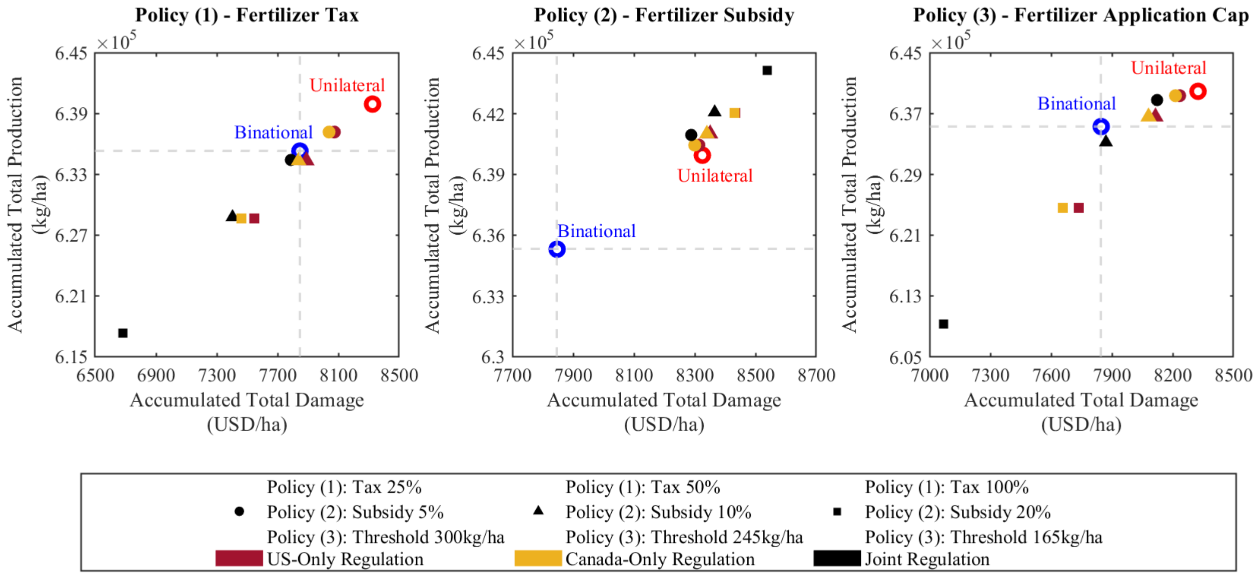

Figure 7: Accumulated Production and Environmental Damage under Unilateral P Policies Figure 7 presents the accumulated total production and environmental damage over a 50-year period based on 10,000 Monte Carlo simulations(The initial level of soil P is the minimum level (i.e., 1 mg/kg)). The results are derived by computing annual averages and summing them over time. For threshold-based policies, the 300 kg/ha limit represents the 90th percentile of the maximum P application observed in our dataset (330 kg/ha), while the 245 kg/ha and 165 kg/ha thresholds correspond to the 75th and 50th percentiles, respectively.

application practices. Application caps impose a hard constraint, limiting fertilizer use regardless of market conditions. Across all three instruments, binational optimization shifts outcomes closer to the social optimum by explicitly incorporating cross-border spillovers.

Figure 7 extends the analysis to unilateral policies, in which each country regulates independently while ignoring external damages imposed on its neighbor. Several important patterns emerge. First, unilateral interventions such as high fertilizer taxes or strict caps can reduce domestic P use and thereby lower environmental damage, sometimes achieving reductions that exceed those in the binational benchmark. However, these gains typically come at higher economic cost, as production falls more sharply compared to cooperative policies.

Second, unilateral subsidies tend to be less effective in mitigating damages, as they are primarily designed to support farmer adoption rather than internalize external costs. Without coordination, subsidies risk shifting production patterns without substantially reducing cross-border spillovers.

Finally, the comparison highlights that the efficiency of binational management does not stem solely from the stringency of regulation but from its scope. By jointly accounting for transboundary nutrient flows, the binational approach aligns incentives such that even moderate interventions yield larger environmental benefits relative to unilateral policies. In other words, the inefficiency of unilateral management arises less from weak policy design and more from the absence of coordination.

These results emphasize that direct controls on fertilizer application can be powerful tools to manage P use, but their effectiveness is highly dependent on the level of cooperation between countries. While unilateral policies can impose reductions through taxes or caps, they often entail larger trade-offs in terms of foregone production.

6.2 Managing Nutrient Losses Beyond Fertilizer Use

A second class of P management policies targets nutrient runoff directly, rather than altering the private costs of fertilizer use. Unlike input-based measures such as taxes, subsidies, or application caps, these policies aim to reduce the amount of P leaving agricultural fields, thereby mitigating eutrophication risk while maintaining crop productivity. Examples include cover crops that capture residual nutrients during non-growing seasons, buffer strips or vegetative filter zones that trap sediment-bound P before it reaches waterways, and mycorrhizal fungi that improve nutrient uptake efficiency in crops (Mayer et al., 2007; Helmers et al., 2008; Won et al., 2024; Kaur et al., 2024; Bromley and Rintoul-Hynes, 2025). By addressing the transport pathway of P, these policies provide an alternative channel for reducing environmental damages without imposing strict limits on fertilizer inputs.

In our modeling framework, phosphorus runoff control is represented as an exogenous reduction in P outflow from agricultural soils, calibrated at three levels: 50%, 60%, and 80% reductions relative to baseline runoff. These scenarios are grounded in empirical findings from agronomic and conservation literature. A study on cereal rye cover crops demonstrated that total P (TP) and dissolved reactive P (PO_4-P) loads were reduced by approximately 70% and 73%, respectively, compared to fields without cover crops (Kaur et al., 2024).

For vegetative filter strips (buffer strips), research shows they can intercept 60%–90% of sediment, reduce runoff volume by 50%–80%, and decrease TP loads by up to 50% (Helmers et al., 2008). These empirical ranges justify the modeling of moderate to high-efficiency runoff-control scenarios—specifically, reductions of 50%, 60%, and 80%—capturing the spectrum from conservative to ambitious mitigation efforts.

Figure 8 presents the outcomes of runoff control policies under three governance regimes: U.S.-only regulation, Canada-only regulation, and joint regulation. The horizontal axis measures

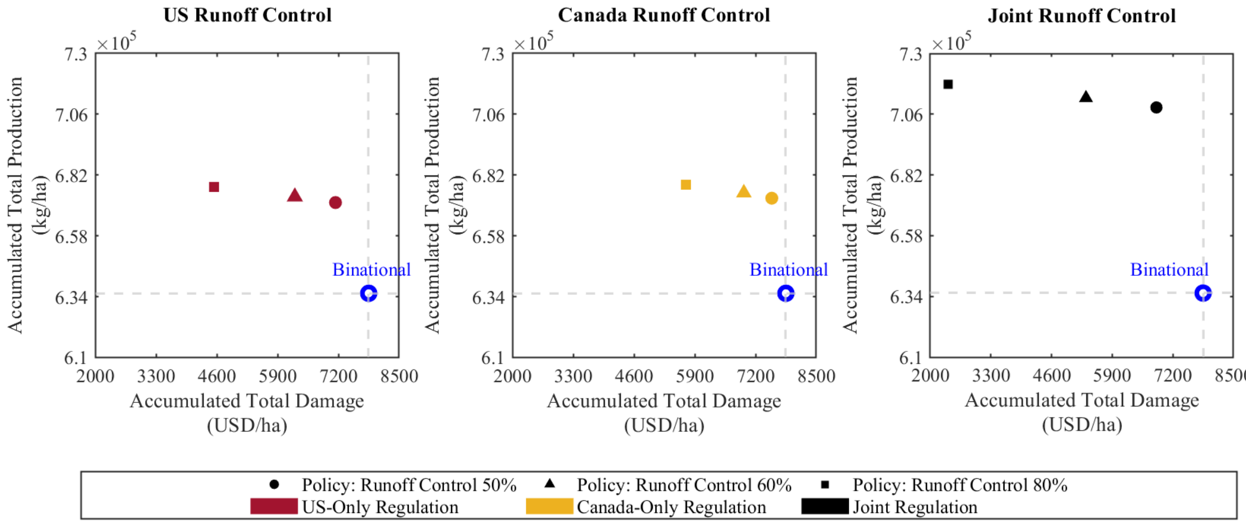

Figure 8: Accumulated Production and Environmental Damage under P Runoff Controls Figure 8 presents the accumulated total production and environmental damage over a 50-year horizon under policies targeting P runoff reduction, based on 10,000 Monte Carlo simulations (initial soil P level = 1 mg/kg). The policy scenarios assume that runoff can be reduced by 50%, 60%, and 80% relative to the baseline, reflecting empirical estimates from the conservation practice literature. The results are derived by computing annual averages of yield and damage and summing them over time.

accumulated total environmental damage (USD/ha), while the vertical axis tracks accumulated total crop production (kg/ha) over the 50-year simulation horizon.

The results reveal several key patterns. First, single runoff control policies generate meaningful reductions in environmental damages, but the magnitude varies by country. Because the U.S. is the dominant source of P loading into shared waterways, U.S.-only policies produce larger damage reductions compared to Canada-only policies. Conversely, Canada-only controls yield limited environmental benefits, since much of the P entering Canadian water bodies originates upstream in the U.S. Nonetheless, Canada-only measures still improve domestic outcomes, particularly when combined with aggressive reductions (e.g., 80%).

Second, joint runoff control policies deliver the most substantial improvements, simultaneously lowering total damages and raising crop production. This dual benefit arises because reducing runoff not only curtails downstream pollution but also retains more P in the soil, thereby enhancing nutrient availability for crops. In contrast to fertilizer taxes or caps, which tend to suppress yields by constraining input use, runoff control policies shift the production-damage frontier outward. The joint regulation results highlight the cooperative gains available when both countries align their efforts to reduce transboundary P spillovers.

A striking implication of these results is that runoff mitigation outperforms direct fertilizer controls along both economic and environmental dimensions. While taxes, subsidies, and caps succeed in reducing damages, they often do so at the expense of crop yields, reflecting a trade-off between agricultural output and environmental quality. By contrast, runoff controls mitigate this trade-off, delivering simultaneous gains in productivity and sustainability.

This outcome can be understood through the lens of the MUC. Input-based policies alter MUC by raising the shadow price of fertilizer, thereby discouraging its application. Runoff policies, however, reduce the environmental damage component of the MUC without diminishing the marginal product of fertilizer. As a result, they shift the balance toward higher net benefits, allowing farmers to maintain or even increase fertilizer use while achieving lower damages. The efficiency advantage of runoff policies lies in their ability to decouple yield gains from pollution damages.

From an economic perspective, the findings suggest that investments in runoff control technologies may offer higher social returns than conventional input regulations. Cover crops, buffer strips, and mycorrhizal amendments not only reduce damages but also enhance long-run soil fertility and resilience. Policies that subsidize or incentivize adoption of such practices may therefore deliver cost-effective environmental benefits while avoiding the production losses often associated with fertilizer taxes or caps.

In turn, runoff control policies represent a highly effective strategy for managing P pollution. By directly targeting the transport of phosphorus into water bodies, such measures reduce environmental damages while enhancing agricultural productivity. The simulation results indicate that reducing runoff by 50–80% yields substantial benefits, especially when implemented jointly across the U.S. and Canada. Compared to traditional input-based policies, runoff controls shift the production-environment frontier outward, delivering superior outcomes with fewer trade-offs. These findings highlight the importance of broadening the policy toolkit beyond direct fertilizer regulation and incorporating ecosystem-based approaches that manage nutrient losses more holistically.

7 Discussion

This study examines the economic and environmental consequences of P management in transboundary systems, such as the Lake Erie basin, and shows how strategic interactions between countries shape long-term agricultural productivity and ecological outcomes. The analysis highlights the inefficiencies of unilateral decision-making, where optimizing solely for domestic objectives overlooks external costs imposed across borders. Such misalignment leads to excessive P use, amplifying runoff and environmental damage beyond socially optimal levels.

A central insight from the dynamic game model is that binational cooperation need not rely on heavy-handed interventions like steep fertilizer taxes or rigid application caps to curb P runoff. Simply incorporating transboundary nutrient spillovers into the optimization framework naturally shifts decisions toward more sustainable P application. In contrast, aggressive unilateral actions, while effective in reducing runoff, often come at the expense of crop productivity, underscoring a persistent trade-off between environmental preservation and economic efficiency.

Policy simulations underscore that both unilateral and cooperative strategies involve trade-offs. While instruments such as fertilizer taxes and caps can reduce environmental damages, they also constrain production, raising concerns for food security and economic resilience. This suggests that P management policies should go beyond restrictive measures and incorporate complementary strategies that sustain yields. Technological innovations—including precision agriculture, enhanced fertilizer efficiency, and soil health practices—offer pathways to reduce nutrient losses while maintaining productivity. Integrating these advancements into policy design could mitigate the efficiency losses of regulation and create win-win outcomes for both agriculture and the environment.

An equally important challenge lies in the unobservability of soil P, especially in transboundary contexts. Farmers face dual uncertainty: (i) incomplete knowledge of their own soil P due to imperfect testing and nutrient cycling, and (ii) uncertainty about neighboring soil conditions, which shape cross-border runoff. While the current model assumes full information, real-world decisions are made under noisy and asymmetric information. Future research should investigate how information asymmetry and learning affect optimal application strategies under binational cooperation. Policies that strengthen soil P monitoring—through improved testing, incentive-based information sharing, or digital soil mapping—could substantially increase the effectiveness of P management in shared systems.

Overall, these findings stress the importance of policies that align economic incentives with sustainability goals. Market-based instruments, such as nutrient trading systems or coordinated subsidies, may provide more flexible and cost-effective approaches than taxes alone. Yet, durable solutions will require pairing economic instruments with investments in technology and monitoring systems that address soil P uncertainty. Incorporating both technological progress and information improvements into binational frameworks offers a promising pathway to achieve sustainable phosphorus management in transboundary agriculture while balancing productivity and environmental quality.

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A Appendix

A.1 Mathematical Proofs and Algorithmic Foundations of Dynamic Game

This appendix provides the mathematical foundations that justify the dynamic game solution algorithm described in the main text. While the main text outlines the implementation of the projection–Newton method, here we establish the contraction property of the Bellman operator, the existence of a stationary Markov Perfect Equilibrium (MPE), the derivation and interpretation of the Marginal User Cost (MUC), the comparative statics of fertilizer use, and the convergence of the algorithm. Together, these results ensure that the computed policies are not only numerical solutions but also theoretically well-grounded equilibrium outcomes.

Fix the opponent's stationary Markov strategy σ_j. The Bellman operator for player i is defined as

$$(T^{\sigma_j}V)(s) = \max_{f_i \in [0,\bar{f}]} \left\{ \mathbb{E}_{\tau_j}[\pi_i(s, f_i)] + \beta\, \mathbb{E}[V(s') \mid s, f_i, \sigma_j(s)] \right\}, \tag{A1}$$

where $s = (l_i, l_j)$ is the state vector.

For any two bounded functions $V, W \in \mathcal{B}(\mathcal{S})$, if $V(s) \leq W(s)$ for all s, then

$$(T^{\sigma_j}V)(s) \leq (T^{\sigma_j}W)(s). \tag{A2}$$

Moreover, for any constant c,

$$(T^{\sigma_j}(V + c))(s) = (T^{\sigma_j}V)(s) + \beta c. \tag{A3}$$

These two conditions—monotonicity and discounting—are exactly Blackwell's sufficient conditions for contraction. Therefore, T^{σ_j} is a contraction mapping on the complete metric space $(\mathcal{B}(\mathcal{S}), \|\cdot\|_\infty)$ with modulus β. By Banach's fixed-point theorem, there exists a unique fixed point $V_i^{\sigma_j}$, which is the value function corresponding to the strategy σ_j. This result justifies the use of value iteration with projection in the algorithm, as convergence to a unique value function is guaranteed (Blackwell, 1965; Stokey and Lucas Jr, 1989; Rust, 1994).

Let Σ_i denote the set of stationary Markov strategies $\sigma_i : \mathcal{S} \to [0, \bar{f}]$. For a fixed $\sigma_j \in \Sigma_j$, the unique value function $V_i^{\sigma_j}$ exists, and the corresponding best response set is

$$\mathcal{BR}_i(\sigma_j) := \arg\max_{\sigma_i \in \Sigma_i} V_i^{\sigma_j}(s). \tag{A4}$$

The best-response correspondence $\mathcal{BR}_i$ is nonempty, convex-valued, and upper hemicontinuous, because the action space is compact and convex, and payoffs and transition functions are continuous. Define the joint best-response correspondence as

$$\mathcal{BR}(\sigma) = \mathcal{BR}_1(\sigma_2) \times \mathcal{BR}_2(\sigma_1), \qquad \sigma = (\sigma_1, \sigma_2). \tag{A5}$$

By Kakutani's fixed-point theorem, there exists $\sigma^* = (\sigma_1^*, \sigma_2^*)$ such that

$$\sigma^* \in \mathcal{BR}(\sigma^*). \tag{A6}$$

This σ^* is a stationary Markov Perfect Equilibrium of the dynamic game (Fudenberg and Tirole, 1991; Maskin and Tirole, 2001).

A.2 Convergence of the Projection–Newton Algorithm

The algorithm described in this section combines value iteration with Newton updates. The contraction property established in A.1 guarantees that value iteration converges to a unique value function for any fixed strategy profile. Given the concavity of the payoff function in fertilizer application, Newton's method converges locally to the unique optimizer. When both countries update their policies iteratively, the best-response mapping satisfies the conditions required for convergence to a fixed point in a neighborhood of equilibrium. This ensures that Algorithm 1 (Dynamic Game Solver) not only converges numerically, but does so to the stationary MPE characterized in A.1. The projection–Newton method therefore provides a reliable computational procedure whose outcomes align with the theoretical structure of the dynamic game (Miranda and Fackler, 2004).

Algorithm 1 Dynamic Game Solver

Input: Model structure $(R_i(s_i, s_j, a_i, a_j), R_j(s_j, s_i, a_j, a_i), g(s_i, s_j, a_i, a_j), \beta)$, initial guesses for $V_i(s_i, s_j)$, $V_j(s_j, s_i)$, policies a_i, a_j, collocation nodes s_i, s_j, tolerance tol, maximum iterations maxit.
Output: Optimal policies a_i^*, a_j^*, value functions $V_i^*(s_i, s_j)$, $V_j^*(s_j, s_i)$.
procedure INITIALIZATION
  Compute collocation nodes for state space s_i and s_j, and the basis function matrix $\Phi(s)$.
  Initialize value functions $V_i(s_i, s_j)$, $V_j(s_j, s_i)$ and control policies a_i, a_j.
end procedure
procedure ITERATIVE VALUE FUNCTION AND POLICY UPDATE
  for each iteration until convergence or maximum iterations **do**
    Step 1: Value Function Update for Player i
    Compute reward $R_i(s_i, s_j, a_i, a_j)$.
    Compute future state $g(s_i, s_j, a_i, a_j)$ and future value function $V_i(s_i', s_j')$.
    Update value function for Player i:

$$V_i(s_i, s_j) \leftarrow R_i(s_i, s_j, a_i, a_j) + \beta \mathbb{E}[V_i(s_i', s_j') \mid s_i, s_j, a_i, a_j]$$

    Step 2: Optimal Control Update for Player i with First-Order Condition
    Solve for the optimal control a_i^* using the Newton method:
    Compute the first derivative (gradient) of the value function with respect to a_i.
    Compute the second derivative (Hessian) of the value function with respect to a_i.
    Update the control a_i using Newton's method:

$$a_i \leftarrow a_i - H(a_i)^{-1} \nabla V(a_i)$$

    Check the first-order optimality condition:

$$\text{Error} = \max(|\nabla V(a_i)|)$$

    if Error $<$ tol **then**
      Converged for Player i; store a_i^*.
    else
      Continue iterations.
    end if
    (Repeat the same steps for Player j): Update value function $V_j(s_j, s_i)$, compute rewards $R_j(s_j, s_i, a_j, a_i)$, and solve for optimal control a_j^* using the Newton method.
  end for
end procedure
procedure OUTPUT
  Return v_i, v_j: Final value functions at evaluation points.
  Return a_i^*, a_j^*: Optimal actions (controls) for both players at evaluation points.
end procedure
Source from Miranda and Fackler 2004

A.3 Additional Figures and Tables

Parameter	Value	Desccription
Biophysical parameters		
μ_η	-0.02	Average depreciation rate (Myyrä et al. 2007)
σ^2	9	Uncertainty variance
δ_1	0.0032	Response parameter of soil P surplus (Ekholm et al. 2005)
δ_2	0.00084	
δ_3	0.000186	Concentration parameter on crop yield (Iho and Laukkanen 2012)
δ_4	0.003	
ρ	1.2	Elasticity of environmental damage to P runoff
r_{US}	0.02	P runoff rate of US farm (Myyrä et al. 2007)
r_{Canada}	0.02	P runoff rate of Canada farm (Myyrä et al. 2007)
$\mathbb{E}[\tau_{\text{US}}]$	0.795	Proportion of US's P loading affecting Canada ($\alpha_{US} = 10$ and $\beta_{US} = 2.572$)
$\mathbb{E}[\tau_{\text{Canada}}]$	0.205	Proportion of Canada's P loading affecting US ($\alpha_{US} = 10$ and $\beta_{US} = 38.78$)
Economic parameters		
β	0.9259	Discount factor with 8% discount rate (Duquette et al. 2012)
p_t^Y	1.737	Corn Price ($ per bushel)
p_t^F	262.357	P fertilizer Price ($ per short ton.)
c	136.5	Marginal cost of P loading (125 €/kg) (Pitkanen et al. 2007)

Table A1: Parameters and descriptions. Corn and P fertilizer prices are from the 2014 prices (USDA 2024a, USDA 2024b) and inflation-adjusted using the using the Consumer Price Index (CPI) for all urban consumers (index 1983=100), with data sourced from the Federal Reserve Bank of Minneapolis 2024.04. The values for $\tau_{\text{Canada}\to\text{US}}$ and $\tau_{\text{US}\to\text{Canada}}$ are calculated as the proportion of P loading from each country relative to the total P loading in a given year (Environment and Climate Change Canada 2023). These yearly proportions are then averaged over the period from 2008 to 2022 to obtain the final values.

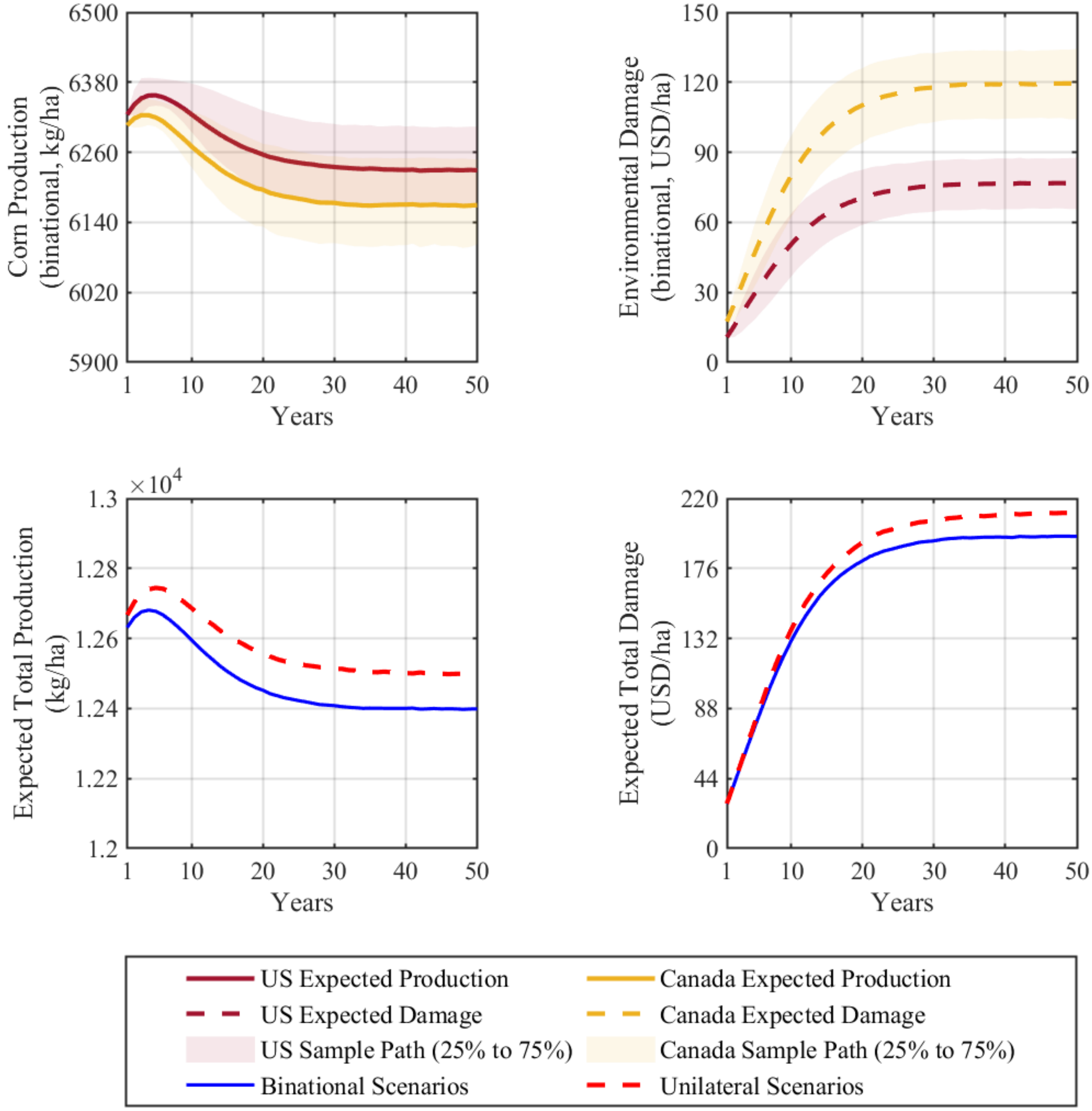

Figure A1: Example of corn production and environmental damage dynamics (Low initial value). The initial level of soil P for the U.S. and Canada is the low level (i.e., 5 mg/kg)

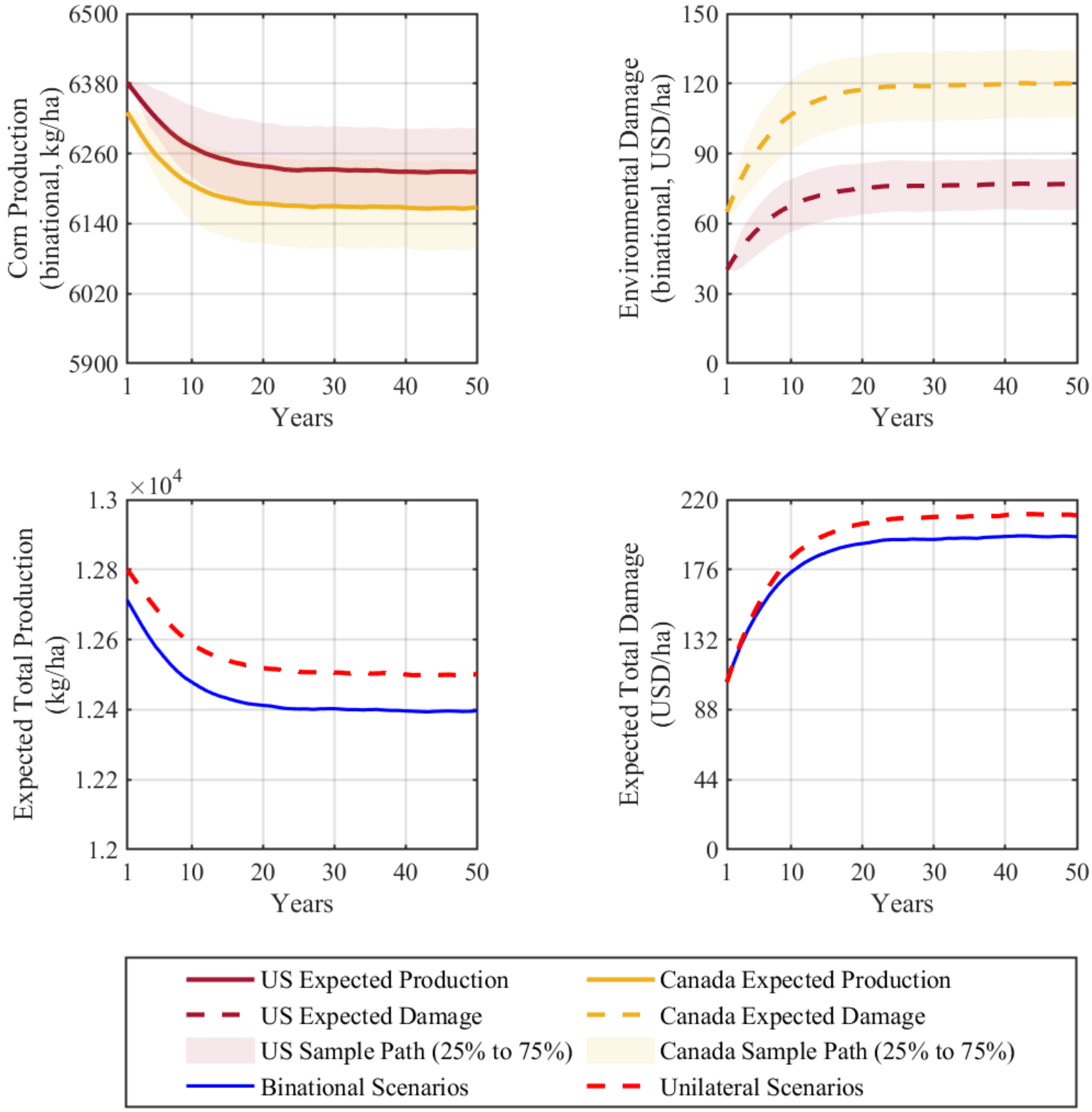

Figure A2: Example of corn production and environmental damage dynamics (Medium initial value). The initial level of soil P for the U.S. and Canada is the medium level (i.e., 15 mg/kg)

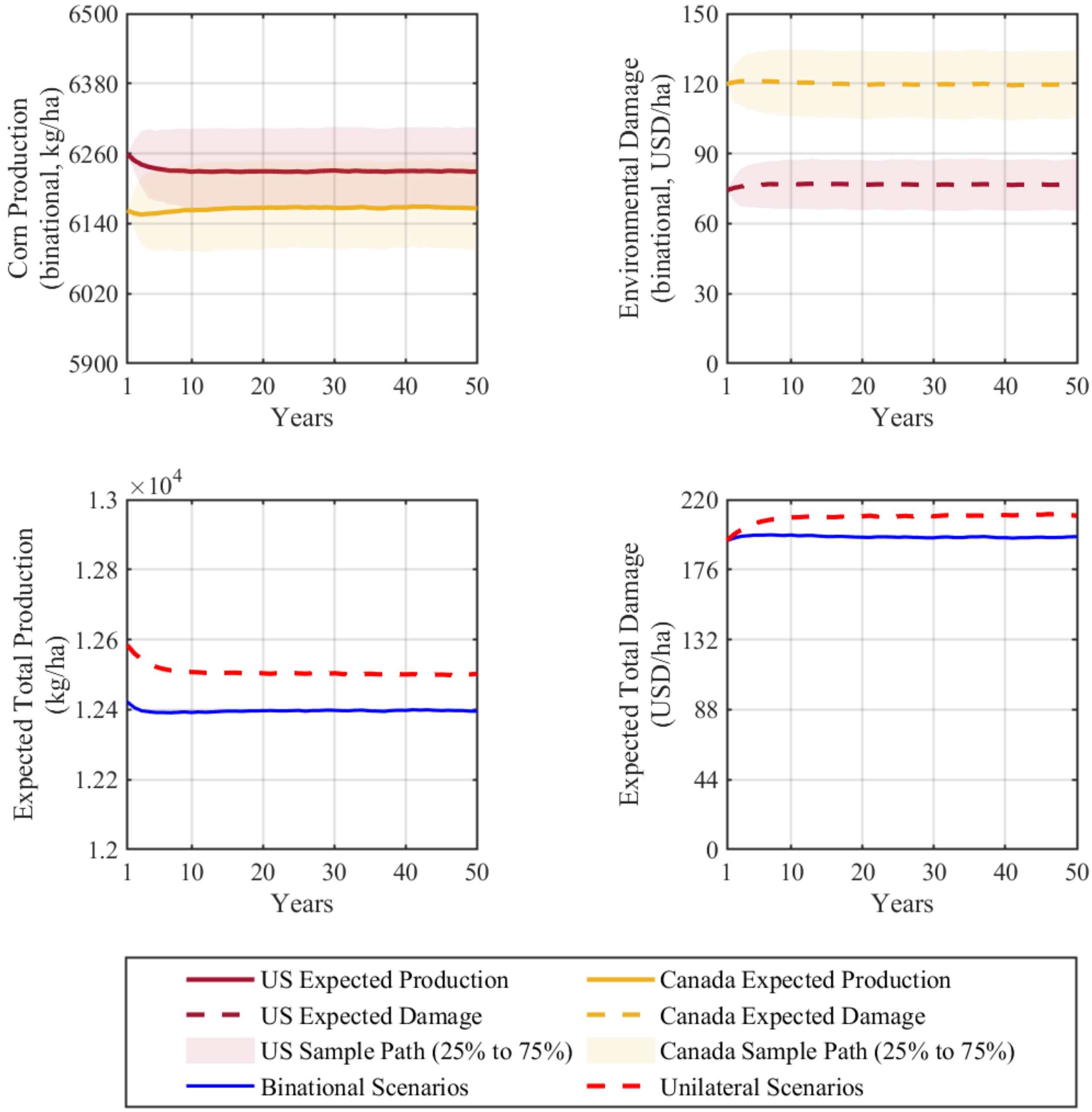

Figure A3: Example of corn production and environmental damage dynamics (High initial value). The initial level of soil P for the U.S. and Canada is the high level (i.e., 25 mg/kg).