Optimal Phosphorus Management in a Transboundary Setting: A Dynamic Game Approach

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March 16, 2025

Abstract

Phosphorus (P) runoff from agriculture is a major driver of eutrophication in transboundary water systems like Lake Erie. This paper develops a dynamic game model to examine how strategic interactions between the U.S. and Canada shape long-term crop production and environmental outcomes under stochastic soil P dynamics. The results show that while unilateral decisions often lead to higher crop production, they also result in greater environmental damage due to excessive P runoff. In contrast, incorporating transboundary nutrient spillovers naturally reduces P application and mitigates environmental harm, though at the cost of lower production. These findings suggest the importance of integrating biophysical feedback and economic incentives in nutrient management, emphasizing that long-term sustainability requires balancing productivity with environmental constraints.

JEL Codes: C73, Q15, Q18, Q53, Q58

Keywords: Transboundary pollution, Binational coordination, Agricultural externalities.

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The authors thank participants at various conferences and seminars for their useful comments and discussions.

Financial Support from the National Science Foundation (NSF) Award CBET-2019435: Science and Technologies for Phosphorus Sustainability (STEPS) Center.

1 Introduction

Lake Erie has long been at the center of discussions on agricultural nutrient management, particularly due to its persistent phosphorus (P) pollution and the resulting harmful algal blooms (HABs) (Smith and Wilen 2003). Excessive P runoff from croplands in the U.S. and Canada has been identified as a primary driver of eutrophication, leading to deteriorating water quality, economic losses in fisheries and tourism, and increased treatment costs for drinking water (Lake Erie LaMP 2011; Environment and Climate Change Canada 2023). Despite decades of policy efforts—including voluntary conservation programs, best management practices (BMPs), and regulatory nutrient reduction targets—P runoff remains a significant challenge, exacerbated by the accumulation of P in soils (Carpenter 2008). The complexity of the Lake Erie case explains the need for dynamic and strategic approaches to P management that account for both long-term nutrient accumulation and transboundary externalities (Brock and Xepapadeas 2010).

While previous studies have examined the economic and environmental trade-offs of P reduction strategies, most rely on static models or single-agent decision frameworks, which fail to capture the strategic interactions between agricultural producers in different jurisdictions (Karp and Zhang 2006). Because P pollution is a transboundary issue, optimal management requires coordinated decision-making between the U.S. and Canada to internalize the spillover effects of nutrient runoff. In the absence of such coordination, unilateral policies may lead to inefficient outcomes, where one country's efforts are offset by the continued externalities imposed by the other (Hoel 1991).

This study develops a dynamic game-theoretic model to analyze optimal P fertilizer application strategies in a transboundary agricultural system, with Lake Erie serving as a motivating case (Xabadia et al. 2008). The model considers the strategic behavior of farmers in the U.S. and Canada, incorporating stochastic soil P dynamics, economic trade-offs between crop production and environmental damage, and cross-border nutrient spillovers (Horan et al. 2019). By solving a Markov decision process (MDP) in a dynamic game setting, we examine how different policy instruments—including fertilizer taxes, subsidies, and application caps—influence long-term environmental and economic outcomes.

A key question addressed in this study is whether unilateral policies—where each country regulates P application independently—can approximate the outcomes of a binationally optimized approach, in which both countries internalize transboundary externalities (Folmer and v Mouche 1994). The findings reveal that while aggressive unilateral interventions (e.g., high fertilizer taxes) can reduce environmental damage, they often come at the cost of reduced crop yields. Conversely, binational coordination accounts for transboundary nutrient spillovers, leading to a more efficient allocation of P fertilizer that reduces environmental damage. However, this also results in lower P application levels compared to unilateral decisions, which may come at the cost of reduced crop production.

By integrating economic incentives, strategic interactions, and transboundary externalities, this study provides a theoretical and quantitative foundation for designing more effective P management policies in shared water systems such as Lake Erie (Gren 2001). The findings explain the importance of cooperative nutrient regulation, focusing on the fact that a combination of policy interventions and technological advancements may be necessary to achieve long-term environmental sustainability without compromising agricultural productivity.

The remainder of the paper is organized as follows. Section 2 provides the background on P pollution in Lake Erie, discussing the role of agricultural runoff and transboundary externalities. Section 3 introduces the dynamic game-theoretic framework, outlining the Bellman equation and the stochastic evolution of soil P levels. Section 4 presents the yield response function estimation, using empirical data from Ohio to quantify the relationship between P fertilizer application and crop yields. Section 5 presents the simulation results, analyzing the economic and environmental trade-offs of different P management policies under unilateral and binational settings. Finally, Section 6 concludes with a discussion on our findings.

2 Background

The Lake Erie basin, straddling the border between the U.S. and Canada, presents a significant environmental management challenge due to P loading, which has profound impacts on water quality, aquatic ecosystems, and economic activities (Lake Erie LaMP 2011; Downing et al.

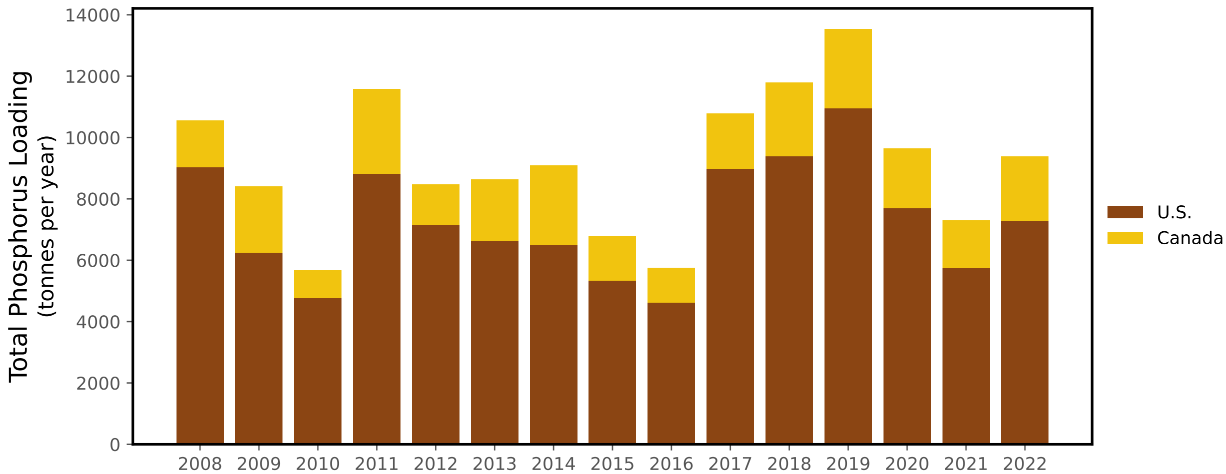

Figure 1: Total phosphorus loading to Lake Erie. The data, sourced from Environment and Climate Change Canada (2023): Canadian Environmental Sustainability Indicators, illustrates the annual P loading into Lake Erie from 2008 to 2022, distinguishing contributions from the U.S. and Canada. The United States consistently accounts for the majority of P loading, contributing over 75% of the total load annually.

2021). Phosphorus is an essential nutrient for plant growth, but when introduced in excessive amounts into freshwater systems, it accelerates eutrophication, leading to the proliferation of HABs (Arrow et al. 2004; Conley et al. 2009; Paudel and Crago 2021; Vasseghian et al. 2024). These blooms can produce toxins harmful to aquatic life, degrade drinking water supplies, and contribute to hypoxic zones (low-oxygen areas) that threaten fish populations and biodiversity.

The sources and magnitudes of P loading to Lake Erie vary across time, space, and jurisdiction, making effective management particularly complex (Scavia et al. 2014; Maccoux et al. 2016). As shown in Figure 1, total P loading to Lake Erie exhibits substantial interannual variability. The U.S. consistently contributes a larger share of total P inputs compared to Canada, with peak loading years. This binational disparity in P contributions has important policy and economic implications. Since Canada contributes a smaller share of total P loading, unilateral mitigation efforts by Canada alone would be costly and inefficient, yielding limited environmental improvements unless matched by reductions in the U.S. watershed. The transboundary nature of P pollution means that any successful reduction strategy requires coordinated efforts between the two nations to avoid cost asymmetries and ensure that the benefits of P reductions are shared equitably. The Great Lakes Water Quality Agreement

Figure 2: Annual average (2013–2022) phosphorus loading patterns and source contributions. The data, sourced from Environment and Climate Change Canada (2023): Canadian Environmental Sustainability Indicators. Figure 2 shows the total P loading into Lake Erie (2008–2022) from multiple sources. Point sources refer to P discharges from municipal sewage treatment plants and industrial effluent, whereas non-point sources primarily stem from agricultural activities and urban stormwater runoff. Atmospheric deposition involves phosphorus settling directly into the lake from the air (Environment and Climate Change Canada 2023).

(GLWQA) provides a framework for such collaboration, setting joint P reduction targets to prevent the burden from falling disproportionately on one country (Loadings and Blooms 2014).

A key challenge in reducing P loading to Lake Erie is the dominance of non-point sources, which account for 77% to 90% of total P inputs across all basins, as shown in Figure 2. While the western basin experiences the highest P loading, non-point sources—mainly from agriculture—are the largest contributors across the western, central, and eastern basins (Environment and Climate Change Canada 2023). Point and atmospheric sources play a relatively minor role, making non-point source management the primary focus for reduction efforts.

Given that non-point source (agriculture) are the primary contributor to P pollution and that P management is inherently a binational challenge, any effective reduction strategy must address both farmers' decision-making and cross-border policy coordination. Since P pollution does not adhere to political boundaries, unilateral efforts are often inefficient and costly, requiring strategic interactions between the U.S. and Canada to achieve shared reduction goals. At the same time, the effectiveness of P mitigation hinges on how farmers adjust their fertilizer use and conservation adoption over time in response to environmental conditions, economic incentives, and policy interventions. Unlike static regulatory approaches, which assume fixed behavioral responses, P management requires a dynamic framework that captures the interactions between policymakers, farmers, and environmental processes across both temporal and spatial scales.

This complexity makes a dynamic game model particularly relevant, as it allows us to analyze how strategic behavior among stakeholders evolves over time. By incorporating economic incentives, uncertainty, and transboundary externalities, the model provides insights into optimal policy coordination between the U.S. and Canada while considering the adaptive nature of agricultural decision-making. In the next section, we develop a dynamic game-theoretic framework to assess how fertilizer application choices, conservation adoption, and regulatory interventions interact, ultimately shaping long-term P loading in Lake Erie.

3 Model

This section introduces the dynamic game model governing soil P accumulation on agricultural land and the resulting economic damages due to soil P runoff. The model captures key processes of the dynamic game modeling approach for the optimal management of soil P, including the carry-over dynamics of soil P, the economic implications of P runoff on farm-level profits, and the stochastic nature of P accumulation and depletion. We first present the transition function of soil P and then extend it to address the resulting runoff and associated economic damages.

3.1 Stochastic soil phosphorus dynamics

We consider a set of farmers denoted by Ψ, each managing their P fertilization strategies. Specifically, for any farmer $i \in \Psi$, the model follows their decisions on the application of P fertilizer over time. The evolution of the stock of soil P, l_{it}, on farmer i's land per hectare at time t, is governed by a dynamic equation that captures both deterministic and stochastic elements. The formulation in this paper builds on the model of Cho et al. 2025.

The evolution of soil P for farmer i is described as:

$$l_{it+1} = \eta_t \left(1 - r_i\right) l_{it} + \left(\delta_1 + \delta_2 l_{it}\right) \underbrace{\left[f_{it} - \overbrace{\left(\delta_3 \log(l_{it}) + \delta_4\right)}^{\text{Concentration on Yield}} y\left(l_{it}, f_{it}\right) \right]}_{\text{Soil P Surplus}} \tag{1}$$

where l_{it} is the stock of soil P at time t, r_i is the P runoff rate to surface water from farm i, and η_t is the stochastic carry-over rate, governing the proportion of soil P that persists from period t to $t+1$. f_{it} represents the amount of P fertilizer applied by farmer i at time t. $y(L_{it}, F_{it})$ is the crop yield function, which depends on both soil P l_{it} and applied P fertilizer f_{it}. $(\delta_1 + \delta_2 l_{it})$ captures the response of soil P surplus to the initial stock level and the rate of P application (Ekholm et al. 2005).

The expression inside the brackets represents the soil P surplus, which is the difference between the P applied through fertilizer f_{it} and the P uptake by crops. Crop uptake is modeled by a yield response function $y(l_{i,t}, f_{i,t})$ scaled by a diminishing return term $(\delta_3 \log(l_{i,t}) + \delta_4)$. This diminishing return effect reflects well-documented agronomic principles, whereby the marginal productivity of P in crop yield decreases as Soil P accumulates (Myyrä et al. 2007; Fulford and Culman 2018; Culman et al. 2023).

The soil P carry-over rate η_t contributes to stochastic motions in this dynamic model. It is formulated to capture the uncertainty in P retention or depletion between periods, and it incorporates both deterministic and stochastic components. Specifically, we model η_t as:

$$\eta_t = \exp\left[\mu_\eta - \frac{s_\eta^2\left(l_{it}\right)}{2} + s_\eta\left(l_{it}\right)\omega_t\right] \tag{2}$$

where μ_η represents the log mean rate of change in soil P, which reflects the natural rate of P retention or decay in the soil. $s_\eta(l_{it})$ is the standard deviation of the log percentage growth rate of soil P, which is modeled as a function of the current stock l_{it}. ω_t is a normally distributed shock term ($\omega_t \sim \mathcal{N}(0, 1)$), which introduces randomness into the carry-over rate, representing external factors such as weather conditions, soil characteristics, or management practices that affect P retention.

The parameter μ_η is assumed to be negative, reflecting the fact that, in the absence of further P application or crop uptake, soil P is expected to decay over time (Ekholm et al. 2005; Iho and Laukkanen 2012). However, this decay process is stochastic, as represented by the inclusion of the standard deviation term $s_\eta(l_{it})\omega_t$. This stochastic component acknowledges that soil P dynamics are influenced by factors beyond the farmer's control, such as soil type, precipitation patterns, and other environmental variables.

The variance of the carry-over rate is specified as:

$$s_\eta^2\left(l_{it}\right) = \ln\left(1 + \frac{\sigma^2}{l_{it}^2 \cdot \mathbb{E}\left[\eta_t \mid l_{it}\right]^2}\right) \tag{3}$$

where σ^2 is an uncertainty variance, and $\mathbb{E}[\eta_t \mid l_{it}]$ represents the expected carry-over rate conditional on the current stock of soil P. This formulation ensures that the variance of the next-period soil P stock remains bounded as l_{it} accumulates, [1] preventing unrealistic behavior where the uncertainty would grow without bound for large l_{it}. Such a specification follows well-established approaches in modeling environmental stocks under uncertainty (Loury 1978, Gilbert 1979; Melbourne and Hastings 2008, Sims et al. 2017; Sloggy et al. 2020).

The introduction of stochasticity in the carry-over process captures real-world complexities where P retention and depletion are not deterministic processes. Factors such as variations in soil composition, temperature, moisture, and microbial activity contribute to the stochastic nature of P dynamics, which this model seeks to represent. By introducing a stochastic component into the P carry-over, the model can better account for observed variabilities in soil P stocks across farms and over time.

3.2 Dynamic game formulation and equilibrium

The strategic interactions between countries Ψ are captured through a Markov perfect equilibrium, which accounts for the fact that each farmer's decision impacts not only their

[1] The expression $\mathbb{E}[\eta_t \mid l_{it}] = \exp(\mu_\eta)$ represents the expected carry-over rate of soil P conditional on the current stock l_{it}. Given the log-normal specification of η_t and assuming the shock term $\omega_t \sim \mathcal{N}(0, 1)$, we can derive the expected value of η_t by taking the conditional expectation with respect to ω_t. Since the expectation of the exponential of a normal random variable ω_t is given by $\exp(s_\eta^2(l_{it})/2)$, the stochastic term cancels out with the variance adjustment term $s_\eta^2(l_{it})/2$, leaving $\mathbb{E}[\eta_t \mid l_{it}] = \exp(\mu_\eta)$. This result implies that the expected value of the carry-over rate is determined solely by the log mean growth rate μ_η, while the variance $s_\eta^2(l_{it})$ introduces uncertainty around this mean, capturing the effects of stochastic shocks.

own payoff but also the runoff and associated damages that affect both players. This approach allows us to study the externalities arising from P runoff and how these externalities influence the optimal management of P fertilization.

The annual payoff of country $i \in \Psi$ is evaluated as the profit generated by crop yields minus the cost of P fertilizer and the damages incurred due to soil P runoff. Formally, the expected per-hectare profit for country i is expressed as:

$$\pi_i \left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}, f_{it} \right) = p^y_{it} y_{it} \left(l_{it}, f_{it} \right) - p^f_{it} f_{it} - d_i \left(l_{it}, \{l_{jt}\}_{j\in\Psi_i} \right) \tag{4}$$

where $\Psi_i = \Psi \setminus \{i\}$ indicates the population Ψ excluding country i. p^y_{it+1} is the price of the crop, p^f_{it} is the price of the P fertilizer, and $y_{it}(l_{it}, f_{it})$ represents the crop yield as a function of the soil P stock l_{it} and the current P fertilizer application f_{it}.

The last term, $d_i \left(l_{it}, \{l_{jt}\}_{j\in\Psi_i} \right)$, represents the damage function, which models the economic damages due to P runoff from both country i's farm and other neighboring country's farms j. Many studies in the literature define the damage function as a linear relationship where environmental (eutrophication) damage is proportional to P runoff, typically expressed as a constant marginal damage parameter multiplied by the amount of soil P runoff (Smith et al. 1995; Sharpley et al. 1996; Iho and Laukkanen 2012, Tang 2018). This approach assumes that each additional unit of P runoff causes the same incremental increase in damage, without accounting for potential threshold effects. However, empirical evidence suggests that eutrophication damage often exhibits nonlinear patterns, where small increases in P runoff may have minimal effects at low concentrations but lead to severe ecological damage once critical thresholds are exceeded (Carpenter et al. 1999; Jarvie et al. 2013, and Schindler et al. 2016). To better capture these dynamics, we define the damage function to a power function with elasticity, given by:

$$d_i(l_{it}, \{l_{jt}\}_{j\in\Psi_i}) = c \cdot \left(r_i l_{it} + \sum_{j\neq i} \tau_j r_j l_{jt} \right)^{\rho} \tag{5}$$

where r_i and r_j represent the runoff rates of soil P from countries i and j, respectively, and

c is the constant marginal damage of P loading. τ is the weight reflecting differences in P transport efficiency between regions. This damage function allows for flexible responses through ρ, where $\rho > 1$ captures threshold effects and $0 < \rho < 1$ reflects diminishing marginal damage.

In the dynamic game setting with multiple countries, the strategic decisions are made over time, considering not only the current payoff but also future consequences of P runoff and its cumulative effects on the environment. Each country optimizes their fertilizer application strategy by weighing the immediate benefits of increased crop yields against the future costs of environmental degradation caused by P runoff. These intertemporal trade-offs are captured by a Markov perfect equilibrium (MPE), where each country's strategy depends only on the current state of the system—specifically, the soil P stocks on both countries, l_{it} and l_{jt}.

The equilibrium concept of the MPE assumes that both countries are noncooperative and forward-looking and that their actions take into account not only the current conditions but also the expected future actions of the other country (Gollier and Treich 2003; Miranda and Fackler 2004; Hovi et al. 2015). The problem is inherently dynamic because the decisions made by each country at time t affect the soil P stock in future periods, which in turn impacts future crop yields and environmental damages. This leads to a situation where both countries must strategically anticipate the other's actions, given the shared nature of the runoff-induced damage.

The optimization problem for each country is framed through the Bellman equation, which represents the recursive nature of the decision-making process. For country i, the value function $V_i\left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}\right)$ reflects the maximum expected net present value (ENPV) of their profits over time, given the current state of soil P stocks on both countries. The Bellman equation for country i is formulated as follows:

$$V_i\left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}\right) = \max_{f_{it}\in\left[0,\bar{f}\right]} \left\{ \begin{array}{l} \pi_i\left(l_{it}, \{l_{jt}\}_{j\in\Psi_i}, f_{it}\right) \\ +\beta\mathbb{E}\left[V_i\left(l_{it+1}, \{l_{jt+1}\}_{j\in\Psi_i}\right) \mid l_{it}, \{l_{jt}\}_{j\in\Psi_i}, f_{it}, \{f_{jt}\}^*_{j\in\Psi_i}\right] \end{array} \right\} \tag{6}$$

where the β is the discount factor and the expectation $\mathbb{E}[\cdot]$ represents the uncertainty in future outcomes, conditional on both countries' current decisions.

The dynamic nature of the problem stems from the fact that each country's decision at time t affects the future states of soil P stocks on both countries, l_{it+1} and l_{jt+1}. Moreover, since P runoff creates externalities that affect both farmers, the value function depends on the current and future decisions of the other country. The term $\{f_{jt}\}^*_{j\in\Psi_i}$ represents the optimal fertilizer application policy of country j, assuming they are also acting optimally given the current state. This interdependence between countrys' decisions is central to the MPE, where each country's strategy is the best response to the other's actions in each period.

To solve for the equilibrium strategies, the system of Bellman equations for the countries must be solved simultaneously. The solution yields the optimal P application policies for both countries, f^*_{it} and f^*_{jt}, which specify the optimal amount of fertilizer to apply in each period, given the current soil P stocks on both countries. These strategies balance the trade-offs between the short-term benefits of increased crop yields and the long-term costs of P runoff.

The MPE ensures that the strategies of both countries are mutually consistent, meaning that neither country has an incentive to deviate from their equilibrium strategy, given the strategy of the other. This equilibrium captures the strategic interdependence between the countries, as each country internalizes the externality caused by P runoff. By following their equilibrium strategies, the countries contribute to managing P runoff in a way that considers not only their own profits but also the broader environmental impacts on the shared ecosystem.

4 Empirical model specification for yield response

4.1 Data description

In this section, we outline the empirical model used to estimate the yield response function y_{it} that analyzes the impact of P fertilizer application and soil P on corn yield. The data used in this estimation originate from long-term field trials in Ohio (Clark, Wayne, and Wood counties) assessing P fertilizer application and their effect on crop yield given soil P levels.

Specifically, we focus on the field trials reported in Culman et al. (2023) Dataset 2, covering 16 years of experiments (2006-2021) at three research farms in Ohio. The trials employed a randomized complete block design with three P application rates: an unfertilized control ($0\times$), an estimated crop removal rate ($1\times$), and an excessive application rate ($2-3\times$ the removal rate) [2] Soil samples were analyzed before planting to determine baseline Mehlich-3 extractable P levels, and crop yield data were recorded after harvest.

For our estimation, we focus exclusively on trials where P fertilizer application resulted in a statistically significant yield increase, excluding non-responsive cases. The dataset thus reflects only instances where P fertilizer had a positive impact on crop yield, ensuring that our estimates capture the actual effect of P application rather than noise from non-significant responses.

Table 1 presents the summary statistics for the Ohio field trial dataset, which includes observations from three experimental locations. The variables reported are corn yield (Mg/ha), P fertilizer application (kg/ha), and soil P concentration (mg/kg). The average corn yield across all sites is 10.23 Mg/ha, with values ranging from 6 to 15 Mg/ha. The mean P application rate is 121.25 kg/ha, with a standard deviation of 126.49 kg/ha, which appears relatively large due to the experimental design. Since this dataset originates from a controlled field experiment with discrete P application treatments ($0\times$, $1\times$, and 2–$3\times$ the estimated crop removal rate) rather than a continuous distribution of fertilizer use, most observations cluster around these predetermined levels rather than being evenly spread across the range. This results in a high standard deviation, as the experimental setup includes both unfertilized plots and excessively fertilized treatments to capture the full range of P fertilizer effects on yield.

4.2 Estimation framework

To quantify the yield response, we specify the following log-linear model:

[2] During the initial phase of the experiment (2006-2014), the estimated crop removal rate for P fertilizer was P_2O_5 60.1kg/ha, based on the estimated removal rate of 2005 Ohio (Vitosh et al. 1995, Fulford and Culman 2018). The field trials consider $2\times$ the removal rate for excessive application cases during this period (Fulford and Culman 2018). However, Fulford and Culman (2018) found that actual removal rates exceeded these estimates. Consequently, from 2015-2021, the fertilizer application rates were adjusted to 112.1kg/ha ($1\times$), and the experiment considered 336.3kg/ha for excessive application cases ($3\times$).

Experimental location	Variable	Obs	mean	Std dev	min/max
Clark	Corn yield (Mg/ha)	30	10.21	2.14	6.2/12.9
	P application (kg/ha)	30	122.66	128.99	0/336.3
	Soil P (mg/kg)	30	20.25	7.88	9.8/40
Wayne	Corn yield (Mg/ha)	36	11.17	2.71	6/15
	P application (kg/ha)	36	127.13	131.29	0/336.3
	Soil P (mg/kg)	36	16.37	9.09	4.3/37.3
Wood	Corn yield (Mg/ha)	48	9.54	2.22	6/14.3
	P application (kg/ha)	48	115.95	123.74	0/336.3
	Soil P (mg/kg)	48	21.28	7.67	11.9/39
Total	Corn yield (Mg/ha)	114	10.23	2.45	6/15
	P application (kg/ha)	114	121.25	126.49	0/336.3
	Soil P (mg/kg)	114	19.46	8.40	4.3/40

Table 1: Summary statistics for Ohio field trials data. The experimental location level is county in Ohio.

$$\ln(y_{it}) = \beta_0 + \beta_1 f_{it} + \beta_2 \ln(l_{it}) + \beta_3 f_{it}^2 + \beta_4 f_{it} \ln(l_{it}) + u_i + \nu_t + \epsilon_{it}^y \tag{7}$$

where u_i is the experimental location fixed effect and ν_t is the time fixed effet. The estimation results are in Table 2. The estimated coefficient on the interaction term ($f_{it} \times \ln(l_{it})$) is -0.0003 with standard error 0.0002; p-value = 0.16). While this coefficient slightly above than the conventional significance thresholds (i.e., p-value = 0.1), it is included in the dynamic game model due to its agronomic and theoretical importance; first, prior agronomic research has emphasized the importance of soil P availability in shaping the response to fertilizer inputs (Fulford and Culman 2018), and second, excluding this term could omit a key mechanism driving farmer decision-making, potentially biasing policy-relevant estimates. Thus, this term remains essential for capturing the full economic and environmental implications of P application.

	Log Corn Yield (Mg/ha)
f_{it}	0.0025***
	(0.0008)
$\ln(l_{it})$	0.1492**
	(0.0602)
f_{it}^2	−0.000003**
	(0.000001)
$f_{it} \times \ln(l_{it})$	−0.0003
	(0.0002)
Const.	1.1959***
	(0.2269)
Location Fixed Effects	Yes
Time Fixed Effects	Yes
Observations	114
Adjusted R^2	0.7810

Table 2: Corn yield response estimation. Robust standard errors are in parentheses. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

5 Results

5.1 Optimal phosphorus application

In our analysis of the Lake Erie case, we simplify the farmer population Ψ to two groups: $\Psi = \{\text{U.S., Canada}\}$. The other parameter values are summarized in Table 3. Given the interconnected nature of agricultural markets and trade between the U.S. and Canada, we assume that both countries share the same prices for P fertilizer and corn. This assumption helps isolate the effects of P runoff dynamics rather than confounding them with price differences, allowing the model to focus on the strategic interactions between farmers in managing P application and the resulting transboundary environmental impacts.

Parameter	Value	Desccription
Biophysical parameters		
μ_η	-0.02	Average depreciation rate (Myyrä et al. 2007)
σ^2	9	Uncertainty variance
δ_1	0.0032	Response parameter of soil P surplus (Ekholm et al. 2005)
δ_2	0.00084	
δ_3	0.000186	Concentration parameter on crop yield (Iho and Laukkanen 2012)
δ_4	0.003	
ρ	1.2	Elasticity of environmental damage to P runoff
r_{US}	0.02	P runoff rate of US farm (Myyrä et al. 2007)
r_{Canada}	0.02	P runoff rate of Canada farm (Myyrä et al. 2007)
τ_{US}	0.795	Proportion of US's P loading affecting Canada
τ_{Canada}	0.205	Proportion of Canada's P loading affecting US
Economic parameters		
β	0.9259	Discount factor with 8% discount rate (Duquette et al. 2012)
p_t^Y	1.737	Corn Price ($ per bushel)
p_t^F	262.357	P fertilizer Price ($ per short ton.)
c	136.5	Marginal cost of P loading (125 €/kg) (Pitkanen et al. 2007)

Table 3: Parameters and descriptions. Corn and P fertilizer prices are from the 2014 prices (USDA 2024a, USDA 2024b) and inflation-adjusted using the using the Consumer Price Index (CPI) for all urban consumers (index 1983=100), with data sourced from the Federal Reserve Bank of Minneapolis 2024.04. The values for $\tau_{\text{Canada}\rightarrow\text{US}}$ and $\tau_{\text{US}\rightarrow\text{Canada}}$ are calculated as the proportion of P loading from each country relative to the total P loading in a given year (Environment and Climate Change Canada 2023). These yearly proportions are then averaged over the period from 2008 to 2022 to obtain the final values.

Figure 3 presents the optimal P application policies for the U.S. and Canada under different soil P conditions and transboundary interactions. The results compare unilateral policies, derived using stochastic dynamic programming (SDP), where each country assumes no transboundary P effect, with binational policies, which account for P spillover across borders. The analysis highlights the inefficiencies in unilateral decision-making and the extent of P misallocation, which exacerbates environmental externalities and results in significant welfare losses for both agricultural producers and environmental stakeholders.

The unilateral P application policy represents the optimal strategy when each country assumes that the other does not contribute to transboundary P levels. In other words, the model optimize P application under the assumption that transboundary P contribution remains constant at zero. This assumption leads to policies that focus solely on domestic soil P levels, disregarding the impact of cross border nutrient flow. However, unilateral policies in shared environmental systems often lead to policy myopia, where short-term gains in productivity come at the cost of long-term environmental degradation.

Conversely, the binational P application policy accounts for the interaction between U.S. and Canada agricultural runoff. When both countries recognize the contribution of transboundary P, the optimal P application rates adjust accordingly, leading to lower application levels as transboundary P increases. The consideration of shared P loads ensures that each country internalizes the externalities of its P use, leading to more environmentally sustainable outcomes. Additionally, this lower application pattern increases when the country has higher domestic P levels.

An important finding of our analysis is the presence of P misallocation (shaded region in Figure 3), where P application under unilateral policies deviate from the binationally optimal levels. This misallocation arises because unilateral policies ignore transboundary P contributions, leading to over application of P fertilizer relative to the socially optimal level. Over-application not only reduces the economic efficiency of fertilizer use but also increases the likelihood of policy intervention in the form of stricter environmental regulations.

For instance, as transboundary P contribution increases, a country adhering to a unilateral policy continues to apply P at the same rate, whereas the binational approach would dictate a reduction in application. This failure to adjust results in excessive P inputs, further contributing to P loading in shared water bodies (e.g., Lake Erie), increasing the risk of eutrophication and HABs. The literature on environmental spillovers suggests that misallocated resources in transboundary pollution settings often generate deadweight losses, where both nations suffer greater costs than necessary due to inefficient policy design (Phaneuf and Requate 2016). Our findings shows the need for cooperative P management policies between the US and Canada to mitigate the environmental consequences of misaligned

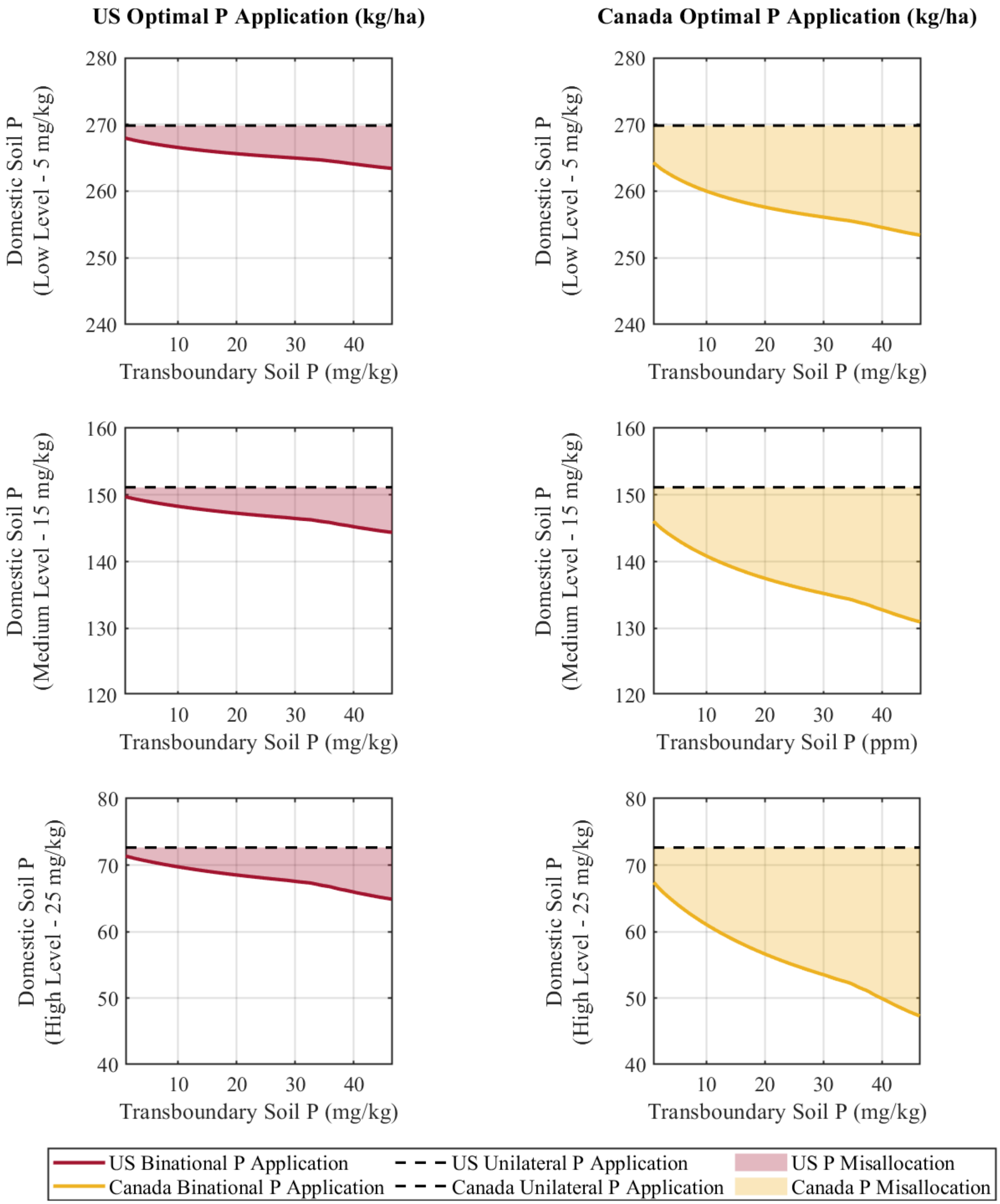

Figure 3: Optimal phosphorus application under domestic and transboundary soil P levels. Domestic soil P level refers to the P concentration within a country's own farmland, affecting its fertilizer needs. Transboundary soil P level represents P levels in a neighboring country, which can influence optimal fertilizer application. For the US, the domestic soil P level (rows) refers to P within the US, while the transboundary soil P level (x-axis) represents P levels in Canada.

agricultural practices.

5.2 Crop production and environmental damage dynamics

Figure 4 presents the long-term evolution of corn production and environmental damage under different P management for the U.S. and Canada. The figure shows differences between U.S. and Canada, as well as the implications of unilateral versus binational P management approaches.

The first row of Figure 4 compares corn production and environmental damage in the U.S. and Canada under binational optimal P application and soil P dynamics. One distinction is that the U.S. receives relatively less external P inflow from Canada than vice versa, due to the directional nature of P runoff as depicted in Figure 1. Because of this asymmetric flow, the U.S. experiences lower external environmental damage from cross-border P spillovers, making the marginal cost of additional P application appear lower. This incentivizes U.S. farmers to apply more P fertilizer, leading to higher soil P levels and greater crop yields compared to Canada. However, due to the directional flow of P runoff, Canada experiences higher long-term environmental damage, as much of the excess P applied in the U.S. This results in greater eutrophication risks and water quality degradation in Canada.

An important implication of this difference is that unilateral strategies, where each country ignores transboundary effects, disproportionately increase the environmental costs borne by Canada. Since U.S. runoff significantly affects Canada but not vice versa, unilateral U.S. policies that fail to account for transboundary nutrient spillovers result in excessive environmental degradation in Canada. This reinforces the need for coordinated binational P management to ensure more sustainable agricultural production in both countries.

The second row of Figure 4 compares total corn production and total environmental damage across both countries under unilateral and binational P application policies. In a unilateral scenario, each country maximizes its own short-term agricultural output without considering cross-border externalities. This results in higher P application levels, leading to greater crop production, as seen in Figure 4 for the unilateral scenario. However, this strategy also leads to substantial long-term environmental damage. Importantly, total environmental damage under the binational scenario is lower than under the unilateral

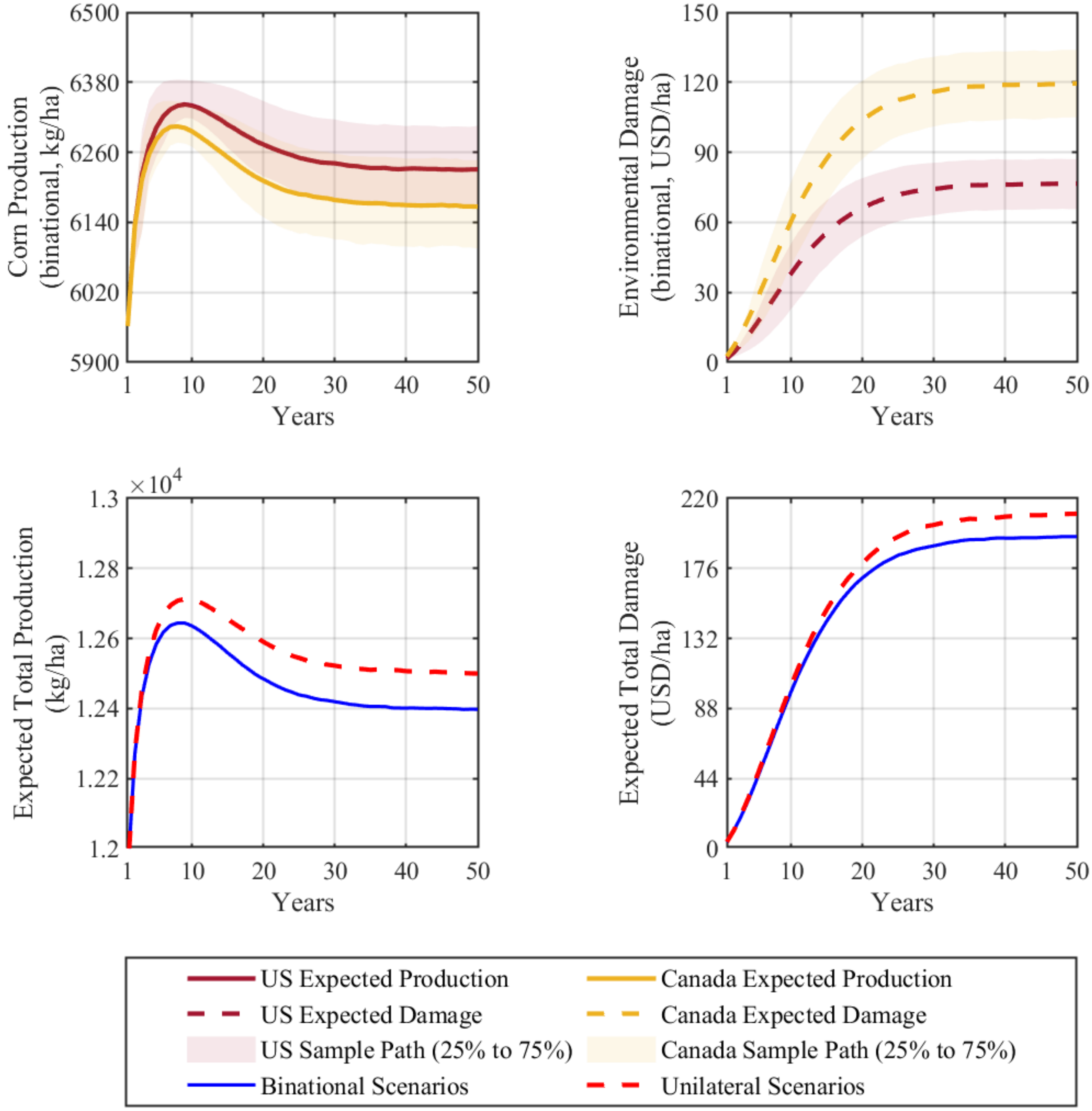

Figure 4: Example of corn production and environmental damage dynamics. Figure 4 presents the simulated trajectories of corn production and environmental damage. Total values represent the sum of the U.S. and Canada cases. The results are based on 10,000 Monte Carlo simulations, with shaded regions indicating the 25% to 75% percentile range of stochastic outcomes. The initial level of soil P for the U.S. and Canada is the minimum level (i.e., 1 mg/kg). Other initial conditions are in the Appendix.

approach, demonstrating that the binational policy suggests the need to consider cross-border P spillovers in optimizing P application and mitigating environmental damage.

These results emphasize the trade-offs between productivity gains and environmental damage in transboundary agricultural systems. Unilateral P application generates more crop yields but leads to excessive environmental damage, necessitating costly regulatory interventions. These results motivate further analysis of potential policy interventions, which are explored in the following section.

5.3 Policy analysis

To evaluate the economic and environmental implications of P management, Figure 5 presents the accumulated total crop production and environmental damage under binational decision-making. This figure illustrates how different policy interventions—fertilizer taxes, subsidies, and application thresholds—affect long-term agricultural productivity and environmental outcomes when applied in a binational optimization framework [3]. Unlike unilateral policies that maximize national objectives without accounting for transboundary effects, the binational approach explicitly incorporates cross-border nutrient spillovers into the optimization process. The results highlight that policies imposing stricter regulations, such as higher taxes or lower application thresholds, generally lead to lower environmental damage but also reduce total crop production.

Next, we extends this analysis by examining the effects of policies under unilateral decision making. Figure 6 presents the accumulated total crop production and environmental damage. Figure 6 explores how unilateral policies—such as fertilizer taxes, subsidies, and application thresholds—affect long-term agricultural productivity and environmental outcomes when applied separately in the U.S., Canada, and jointly. A key question is whether certain unilateral policies can approximate the outcomes of a binationally optimized P management strategy, in which each country internalizes transboundary effects in its decision-making

[3]For clarity, we refer to these price increases as *ad valorem* taxes and subsidies and adjust the fertilizer price accordingly. Specifically, the taxed fertilizer price is defined as $p_{it}^{f,\text{tax}} = p_{it}^{f,\text{tax}} \cdot (1 + \text{Tax Rate})$ where p_{it}^{f} represents the producer price, and p_{it}^{f} denotes the effective price farmers pay after taxation. Similarly, the subsidized fertilizer price is computed as $p_{it}^{f,\text{sub}} = p_{it}^{f} \cdot (1 - \text{Subsidy Rate})$ where $p_{it}^{f,\text{sub}}$ represents the reduced price farmers pay after applying the subsidy.

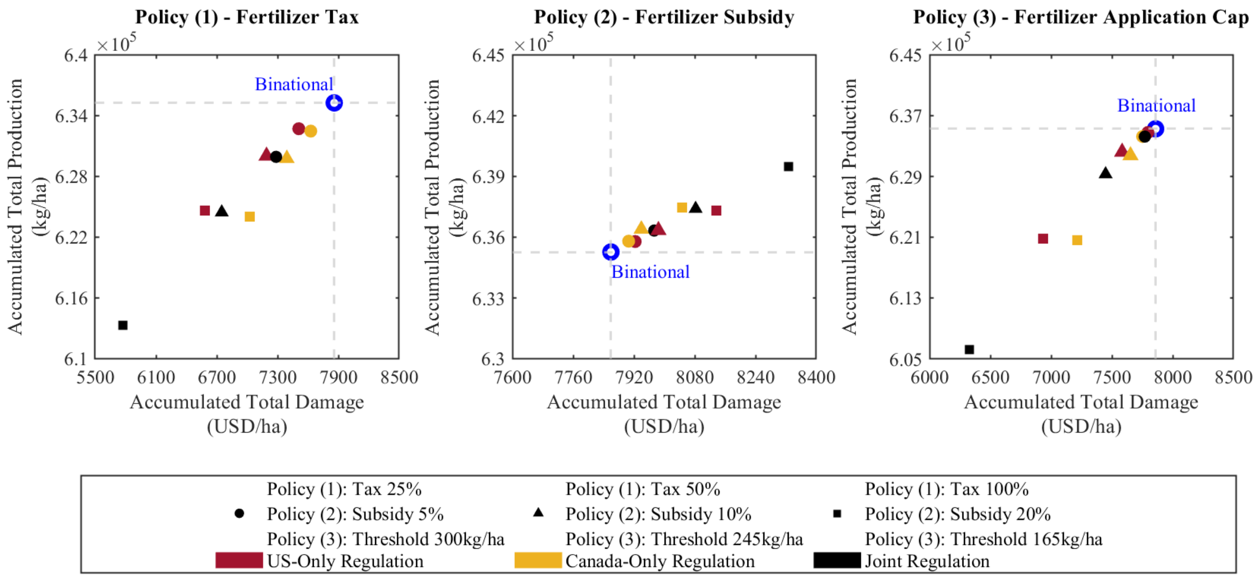

Figure 5: Accumulated production and environmental damage under P policies Figure 5 presents the accumulated total production and environmental damage over a 50-year period based on 10,000 Monte Carlo simulations(The initial level of soil P is the minimum level (i.e., 1 mg/kg)). The results are derived by computing annual averages and summing them over time. For threshold-based policies, the 300 kg/ha limit represents the 90th percentile of the maximum P application observed in our dataset (330 kg/ha), while the 245 kg/ha and 165 kg/ha thresholds correspond to the 75th and 50th percentiles, respectively.

process.

The results indicate that aggressive unilateral policies, such as high fertilizer taxes or strict application caps, lead to both lower crop yields and reduced environmental damage, in some cases achieving even greater reductions in P runoff than the binational benchmark. This suggests that stringent regulation at the national level can effectively curb environmental externalities, though often at the expense of economic output. However, the defining characteristic of a binational approach is not necessarily the direct imposition of strict regulations, but rather the incorporation of cross-border nutrient spillovers into optimal decision-making. Unlike unilateral policies, which maximize national objectives without considering transboundary effects, a binational strategy explicitly accounts for how one country's actions influence the other.

A notable insight from these findings is that simply incorporating the externalities associated with P runoff into each country's optimization problem—without imposing any additional policy interventions—naturally leads to lower environmental damage. In other

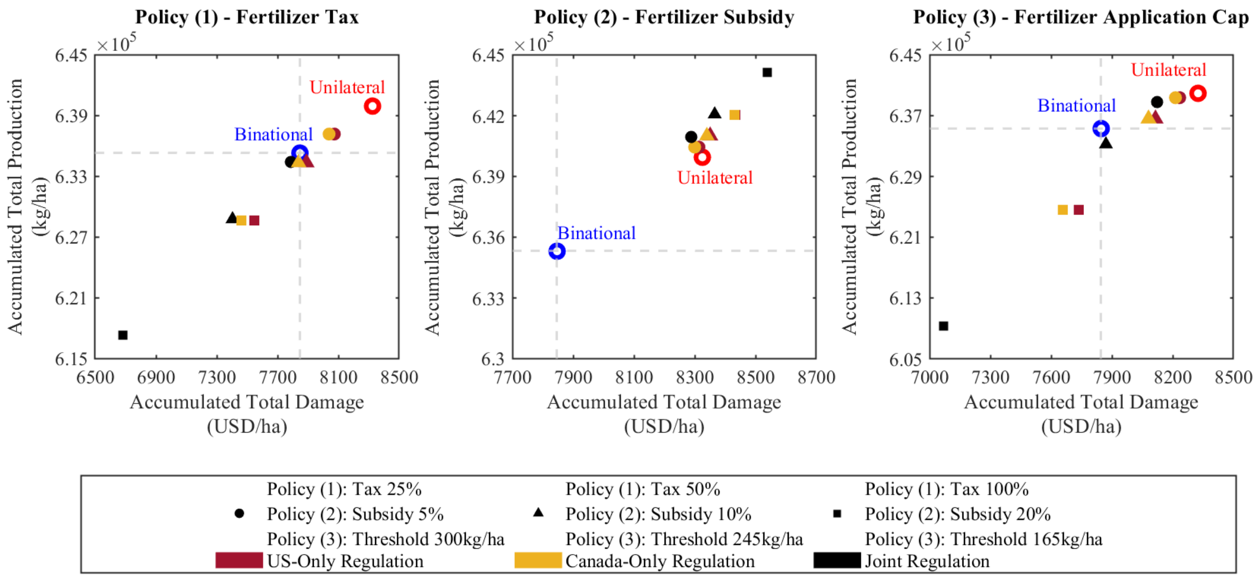

Figure 6: Accumulated Production and Environmental Damage under Unilateral P Policies Figure 5 presents the accumulated total production and environmental damage over a 50-year period based on 10,000 Monte Carlo simulations(The initial level of soil P is the minimum level (i.e., 1 mg/kg)). The results are derived by computing annual averages and summing them over time. For threshold-based policies, the 300 kg/ha limit represents the 90th percentile of the maximum P application observed in our dataset (330 kg/ha), while the 245 kg/ha and 165 kg/ha thresholds correspond to the 75th and 50th percentiles, respectively.

words, if each country were to adjust its fertilizer application while accounting for cross-border spillovers, the resulting P use decisions would already lead to a more sustainable outcome. This suggests that the environmental inefficiency in unilateral P management arises from the lack of coordination rather than the absence of strict policies. While unilateral policies can force reductions in environmental damage through taxation or application limits, a binational approach achieves similar or better outcomes by aligning incentives without necessarily resorting to heavy-handed interventions.

These findings underscore the importance of designing P management strategies that facilitate cross-border coordination, rather than relying solely on unilateral regulatory mechanisms. Policies that encourage farmers to internalize transboundary effects—whether through cooperative agreements, information-sharing, or incentive-based mechanisms—could achieve significant environmental benefits without imposing excessive costs on agricultural production.

6 Discussion

This study explores the economic and environmental damage associated with phosphorus P management in a transboundary setting, such as Lake Erie, demonstrating how strategic interactions between countries influence long-term agricultural productivity and environmental outcomes. The results show the inefficiencies of unilateral decision-making in managing P runoff, where countries optimizing solely for domestic objectives fail to account for the external costs imposed on their neighbors. This misalignment leads to excessive P application, exacerbating environmental damage beyond socially optimal levels.

A key insight from the dynamic game model is that binational cooperation does not necessarily require imposing stringent regulatory interventions, such as high fertilizer taxes or strict application caps, to achieve lower environmental damage. Instead, the mere act of incorporating transboundary nutrient spillovers into the optimization process naturally leads to more sustainable P application decisions. In contrast, aggressive unilateral policies, while capable of reducing runoff, often do so at the cost of lower agricultural productivity, indicating a fundamental trade-off between environmental preservation and economic efficiency.

The policy simulations further reveal that both unilateral and binational approaches involve trade-offs. While fertilizer taxes and application caps effectively reduce environmental damage, they also constrain crop production, raising concerns about long-term food security and economic viability. This suggests that policy interventions should not only focus on reducing P runoff but also consider complementary strategies to sustain or enhance agricultural productivity. Technological innovations, such as precision agriculture, improved fertilizer efficiency, and soil health management, could mitigate the negative effects of regulatory policies by maintaining yields while minimizing nutrient losses. Future research should explore how integrating these advancements into P management frameworks could achieve both environmental and economic objectives simultaneously, reducing the need for strict regulatory interventions that inherently limit productivity.

Another important aspect that warrants further investigation is the unobservability of soil P levels, particularly in a transboundary context. Farmers face two layers of uncertainty: (i) uncertainty regarding their own soil P levels due to imperfect soil testing and nutrient cycling processes, and (ii) uncertainty regarding their neighbor's soil P status, which affects cross-border runoff and environmental damage. The current model assumes that decision-makers have full knowledge of soil P conditions, but in reality, such information is often incomplete or noisy. Future work should explore how information asymmetry and learning mechanisms affect optimal P application decisions, particularly under binational coordination. Developing policies that enhance soil P monitoring—such as improved sampling techniques or incentive-based information-sharing mechanisms—could significantly improve the effectiveness of P management strategies in transboundary agricultural systems.

Overall, these findings emphasize the importance of designing policies that align economic incentives with environmental sustainability. Market-based instruments, such as nutrient trading programs or regionally coordinated subsidy schemes, could provide a more efficient pathway for managing P runoff while preserving agricultural productivity. However, policies that solely rely on economic disincentives, such as taxes, may not be sufficient in the long run without parallel investments in technology-driven solutions that enhance production efficiency. Addressing soil P unobservability, particularly the dual challenges of self-monitoring and cross-border information asymmetry, is critical for ensuring that P management strategies remain both effective and adaptable under real-world conditions. Future research should examine how these policy instruments can be optimally combined with technological advancements and improved information systems to achieve sustainable phosphorus management in shared agricultural systems.

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A Appendix

A.1 Dynamic Game Algorithm

We use the algorithm from Miranda and Fackler (2004) to solve a dynamic game of phosphorus (P) fertilizer management between two interacting countries, the U.S. and Canada. Algorithm 1 iteratively solves the Bellman equation using a projection method to approximate the value function and determines the optimal P fertilizer application policy for each country. Starting with initial guesses for the value function and control policy, the algorithm updates both by evaluating the reward and transition functions at collocation points in the state space [4] . Newton's method is employed to refine the control policy by minimizing the Bellman equation residuals using the gradient and Hessian of the value function. This process is repeated until convergence, ensuring that each country's optimal policy internalizes the externalities of P runoff from the other farm, leading to strategic interdependence in decision-making.

[4] The initial guess for the value function and control policy is obtained from solving a single-agent stochastic dynamic programming (SDP) with a 100-year time horizon. This provides a reasonable benchmark for the starting values in the dynamic game framework.

Algorithm 1 Dynamic Game Solver

Input: Model structure $(R_i(s_i, s_j, a_i, a_j), R_j(s_j, s_i, a_j, a_i), g(s_i, s_j, a_i, a_j), \beta)$, initial guesses for $V_i(s_i, s_j)$, $V_j(s_j, s_i)$, policies a_i, a_j, collocation nodes s_i, s_j, tolerance tol, maximum iterations maxit.
Output: Optimal policies a_i^*, a_j^*, value functions $V_i^*(s_i, s_j)$, $V_j^*(s_j, s_i)$.
procedure Initialization
  Compute collocation nodes for state space s_i and s_j, and the basis function matrix $\Phi(s)$.
  Initialize value functions $V_i(s_i, s_j)$, $V_j(s_j, s_i)$ and control policies a_i, a_j.
end procedure
procedure Iterative Value Function and Policy Update
  for each iteration until convergence or maximum iterations **do**
    Step 1: Value Function Update for Player i
    Compute reward $R_i(s_i, s_j, a_i, a_j)$.
    Compute future state $g(s_i, s_j, a_i, a_j)$ and future value function $V_i(s_i', s_j')$.
    Update value function for Player i:

$$V_i(s_i, s_j) \leftarrow R_i(s_i, s_j, a_i, a_j) + \beta \mathbb{E}[V_i(s_i', s_j') \mid s_i, s_j, a_i, a_j]$$

    Step 2: Optimal Control Update for Player i with First-Order Condition
    Solve for the optimal control a_i^* using the Newton method:
    Compute the first derivative (gradient) of the value function with respect to a_i.
    Compute the second derivative (Hessian) of the value function with respect to a_i.
    Update the control a_i using Newton's method:

$$a_i \leftarrow a_i - H(a_i)^{-1} \nabla V(a_i)$$

    Check the first-order optimality condition:

$$\text{Error} = \max(|\nabla V(a_i)|)$$

    if Error $<$ tol **then**
      Converged for Player i; store a_i^*.
    else
      Continue iterations.
    end if
    (Repeat the same steps for Player j): Update value function $V_j(s_j, s_i)$, compute rewards
    $R_j(s_j, s_i, a_j, a_i)$, and solve for optimal control a_j^* using the Newton method.
  end for
end procedure
procedure Output
  Return v_i, v_j: Final value functions at evaluation points.
  Return a_i^*, a_j^*: Optimal actions (controls) for both players at evaluation points.
end procedure
Source from Miranda and Fackler 2004

A.2 Additional Figures

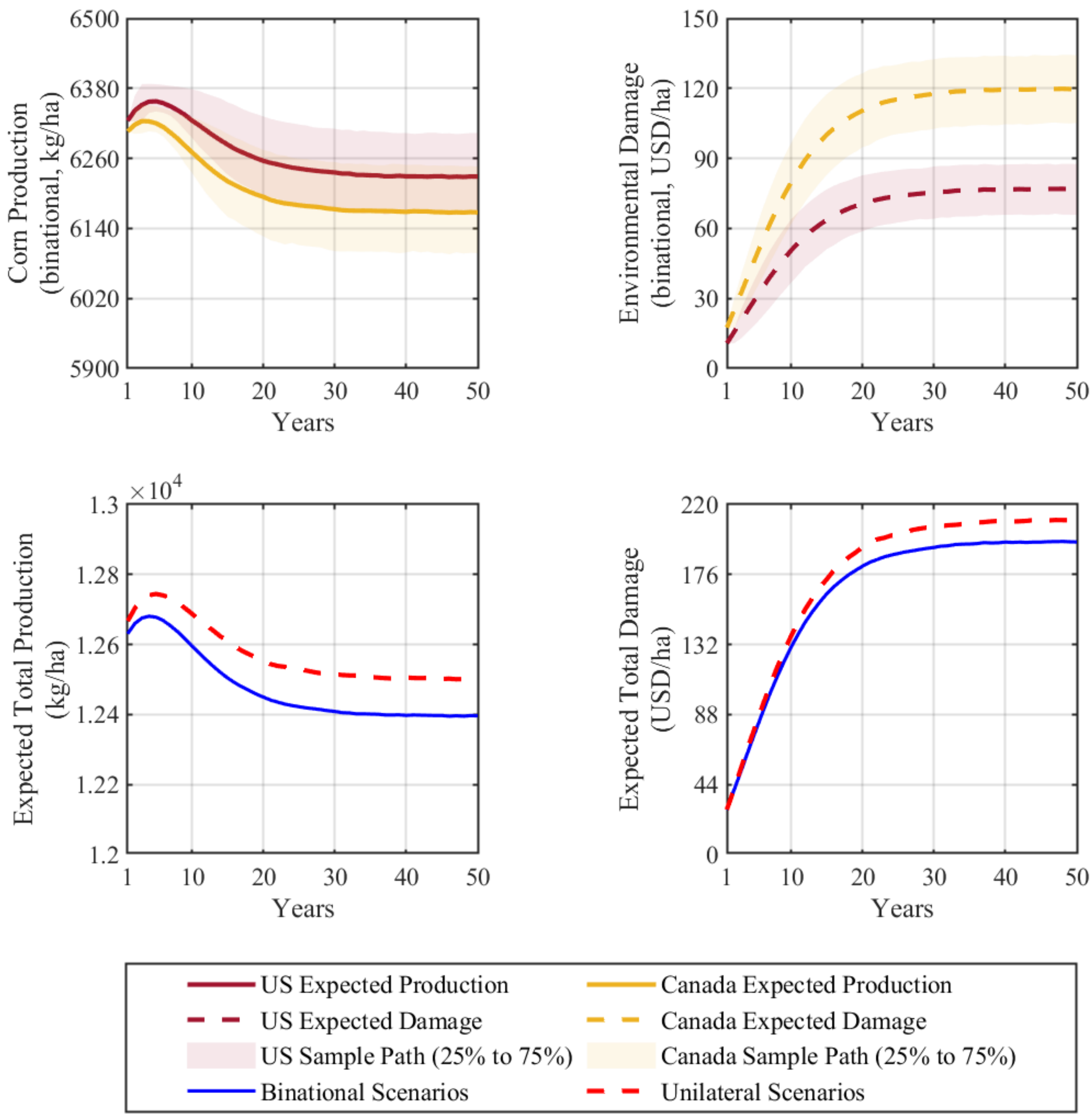

Figure A1: Example of corn production and environmental damage dynamics (Low initial value). The initial level of soil P for the U.S. and Canada is the low level (i.e., 5 mg/kg)

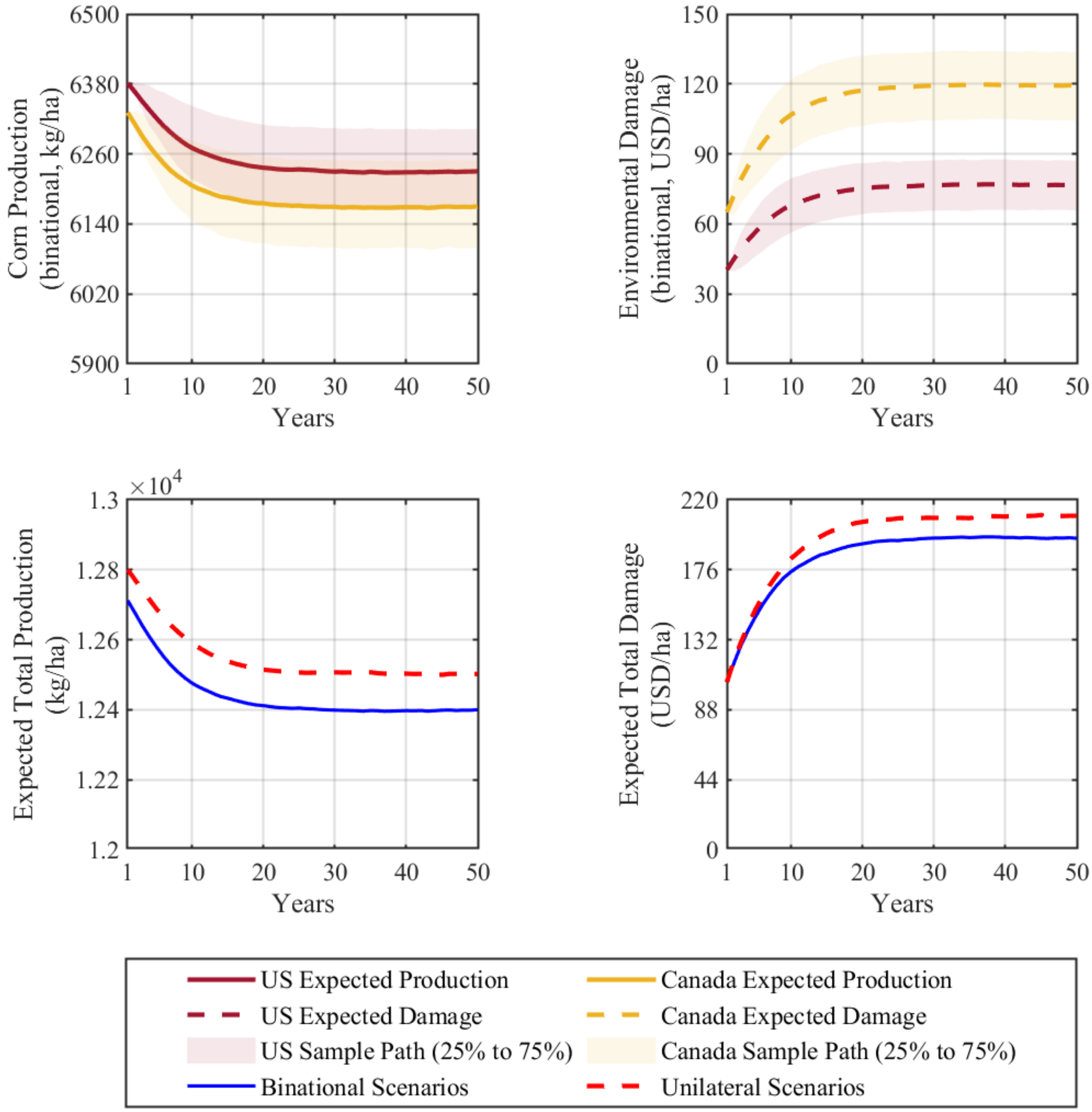

Figure A2: Example of corn production and environmental damage dynamics (Medium initial value). The initial level of soil P for the U.S. and Canada is the medium level (i.e., 15 mg/kg)

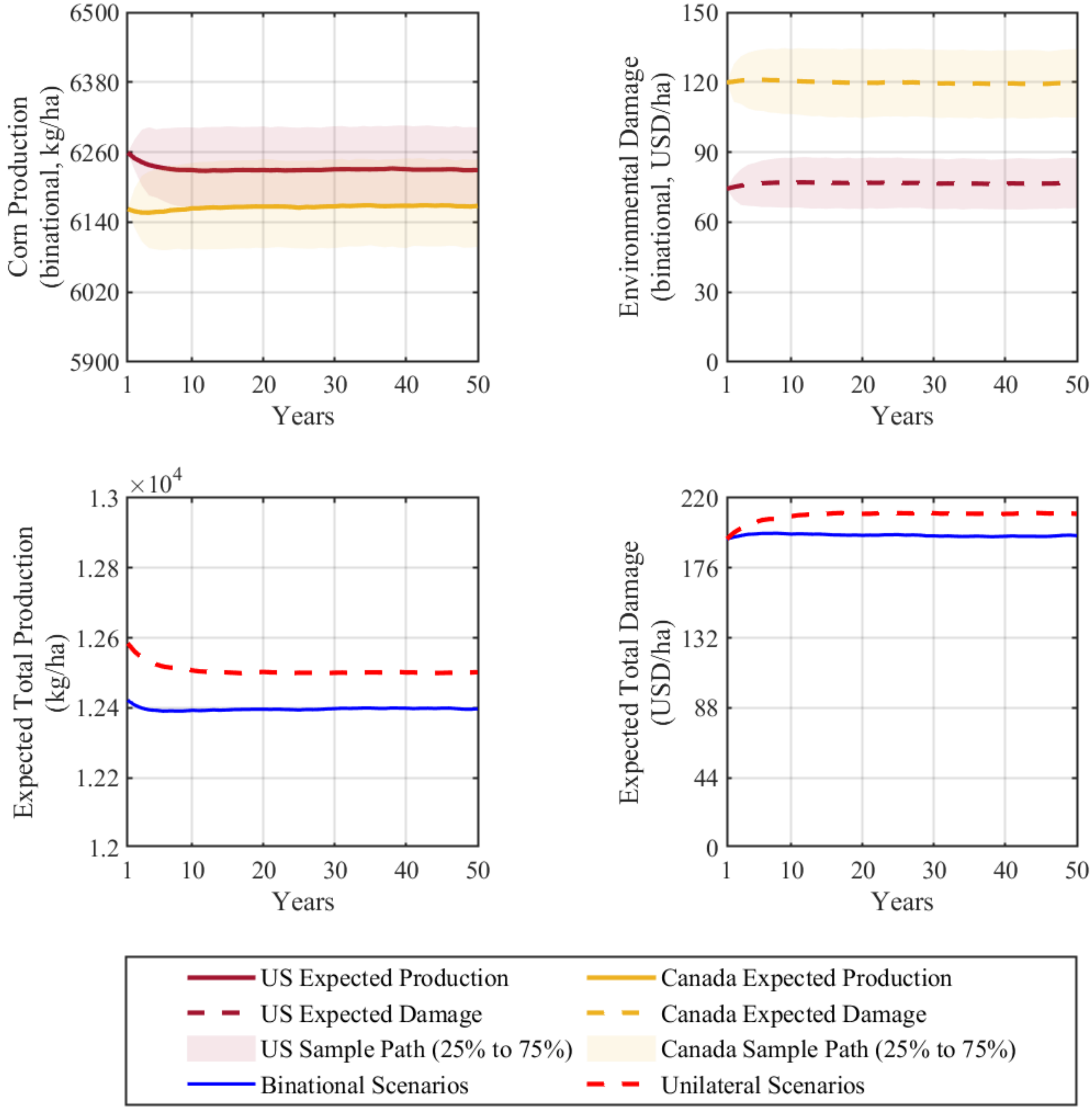

Figure A3: Example of corn production and environmental damage dynamics (High initial value). The initial level of soil P for the U.S. and Canada is the high level (i.e., 25 mg/kg).